WAS CLASSICAL LOGIC SUNK IN ARISTOTLE'S SEA BATTLE?
Timothy Cleveland

Logic sometimes leads to absurdities. Since absurdities are unacceptable, if not downright incoherent, something's got to give. Suspicion is best placed on the premises when such arguments arise. The more self-evident or innocuous the premises, however, the more the pressure mounts on the logical principles themselves. With enough pressure logic itself will break. The laws of logic are not sacrosanct. This theme has been familiar since Quine's "Two Dogmas of Empiricism." According to Quine, some apparently absurd scientific results might be so well confirmed that the only reasonable way out would be to jettison certain so-called "laws" of logic:

> Even a statement very close to the periphery (i.e., an observation
> statement) can be held true in the face of recalcitrant experience
> by pleading hallucination or by amending certain statements of
> the kind called logical laws. Conversely, no statement is immune
> to revision. Revision even of logical law of excluded middle has
> been proposed as a means of simplifying quantum mechanics . . .[1]

This attitude toward the status of logic can be called *logic naturalized*, placing logic on the same epistemological footing as science. But one needs no knowledge of modern science to think logic better off without the law of excluded middle. All one may need is the conviction that sometimes deliberation matters, that not everything that happens, happens of necessity. This metaphysical conviction was the force behind Aristotle's sea battle argument. The challenge Aristotle makes is that if the principle of bivalence holds or if the law of excluded middle is true, then fatalism inescapable—everything happens of necessity and deliberation is futile. The metaphysical conviction—that all is not necessary—is so strong that the commitment to bivalence and the law of excluded middle must be abandoned. A new logic seems necessary. Set science and the mysteries of quantum physics aside. Classical logic appears to have been doomed since classical times. Aristotle's sea battle apparently proves the need for alternatives to classical logic based on bivalence and the law of excluded middle. Avoiding fatalism may demand the adoption of some modal or many-valued logic. There seems no choice in the matter. But what seems so is not necessarily so. Aristotle's ancient reasoning, despite appearances to the contrary, does not vindicate the modern interest in modal and many-valued logic. Classical logic may or may not survive quantum mechanics, but it was not sunk in the ancient sea battle.

Aristotle's reasoning is easy enough to summarize though the details are notoriously difficult. Assume the principle of bivalence—that every statement is either true or false. This assumption ensures the necessity of the law of excluded middle—for any sentence p, either p or not-p. Consider the future tense sentence "There will be a sea battle tomorrow." Necessarily, either it or its negation is true. Suppose today someone claims, "There will be a sea battle tomorrow." If the day arrives and there is a sea battle, then he *was* right all along. The sentence *must* be true or false today, and it certainly is not false because things happen

just as it says. So, it is *impossible* for it to be false; there are no other possibilities. If it is *impossible* that there *not* be a sea battle tomorrow, then necessarily, there will be a sea battle tomorrow. Nothing one can do can make it otherwise. Therefore, fatalism is inevitable, everything happens of necessity and deliberation is useless.

Such a summary certainly shows what is at stake, but any evaluation demands a more careful analysis. The original source is Aristotle's *De Interpretatione* 9.[2] Early in chapter 9 he says, "For if every affirmation or negation is true or false it is necessary for everything either to be the case or not be the case" (18ᵃ 34-35). Aristotle is apparently claiming that if bivalence holds then the law of excluded middle is a necessary truth:

> For every statement *p*, *p* is True or *p* is False. [Principle of Bivalence]

> So, *necessarily*, either *p* or not-*p*. [Law of Excluded Middle]

The force of the fatalism may seem to come from the necessity of the law of excluded middle. But this modal claim is misleading. What Aristotle says is ambiguous. There are two possibilities depending on the scope of the necessity:

> A. If bivalence holds, then the law of excluded middle is necessary.

In this case, what follows from bivalence is a necessary truth. The necessity attaches to the law of excluded middle. (A) i**s** a stronger modal claim than:

> A. Necessarily, if bivalence holds, then the law of excluded middle is true.

This necessity is simply the logical necessity of one proposition implying another. On this interpretation, Aristotle would not be attaching any special necessity to the law of excluded middle. He would simply be asserting a necessary connection between two claims. This necessity is what Aristotle called "hypothetical."[3] If that were the correct reading of Aristotle's claim, then it would be wrong to find the necessity of the future in the *necessity* of the law of excluded middle, as though some special necessity in the law of excluded middle simply transferred to the necessity of true future tense statements. The (A) interpretation should be avoided because it suggests that Aristotle commits a simple-minded modal fallacy:

> *Necessarily,* either *p* or not-*p*.

> Suppose, the future tense sentence *p* is true.

> *Necessarily*, *p* is true.

Whatever mistakes Aristotle might be making, this is surely not one of them. He is not simply distributing necessary over a disjunction. If one assumes Aristotle means (A), one is tempted to foist this fallacy on him. Aristotle's language does explicitly give the modal operator narrow scope. He does attach the necessity directly to the law of excluded middle. But this stylistic evidence is not conclusive.

Perhaps Aristotle did intend (A), but that assumption is unnecessary. (A) is much stronger, than (B), but (B) is all Aristotle needs for his argument. Charity will foster understanding here by focusing us on what really is at issue. All that an understanding of Aristotle's text demands is that the argument begins with the following syllogism.

1. Necessarily, if bivalence holds, then for all *p*, either *p* or not-*p*. [Logical Implication]
2. <u>Assume bivalence holds.</u>
3. For all *p*, either *p* or not-*p*.

Aristotle is simply saying the law of excluded middle follows necessarily from the principle of bivalence. What Aristotle recognized is as simple as the truth-table for "*p* or not-*p*":

p	~*p*	*p* v ~*p*
T	F	T
F	T	T

Given bivalence—that every sentence is true or false—these are all the possibilities, and it is obvious that there is no possibility in which the principle of excluded middle is false. Thus given bivalence, the law of excluded middle is necessary. That is merely to say that it is a tautology however. From this tautology—together with a substantial modal claim—fatalism is supposed to follow.

Bivalence implies the law of excluded middle. How does the law of excluded middle lead to fatalism? Given the law of excluded middle, either there will be a sea battle tomorrow or there will not be a sea battle tomorrow. One will occur, but why *must* that one occur? One and not the other will happen, but why is it, and not the other, *bound* to happen? Aristotle needs another modal premise.

He finds his principle in the connection between time and necessity. Whatever is always true is necessary. With this modal principle he can then make his argument. Consider the statement "There will be a sea battle tomorrow." If the sea battle happens, then this statement *was* true today and *would have been* true if, as Aristotle puts it, someone had ". . . said ten thousand beforehand that this would be the case" (*De Interpretatione* 18^{b}34-35). Aristotle then points out if that is true, then the statement has always been true: ". . . if it is white now it was true to say earlier that it would be white; so that it was always true to say of anything that has happened that it would be so. But if it was always true to say that it was so, or would be so, it could not not be so, or not be going to happen" (*De Interpretatione* 18^{b}10-14). Aristotle then states the undeniable modal truth of necessity/possibility interchange: "But if something cannot not happen it is impossible for it not to happen; and if it is impossible for something not to happen it is necessary for it to happen" (*De Interpretatione* 18^{b}13-14). He then reaches the devastating conclusion, "Everything that will be, therefore, happens necessarily" (*De Interpretatione* 18^{b}15.

Aristotle's argument can now be spelled out clearly.
1. Assume every statement (and its negation) is true or false. [Bivalence]
2. Necessarily, if every statement is true or false, then for any *p*, either *p* or not-*p*.
3. Therefore, for any *p*, either *p* or not-*p*. [Law of Excluded Middle]
4. Consider the claim "There will be a sea battle tomorrow."
5. This claim is true or false. (from 3)

6. If tomorrow comes and it *is* true, then it *was* true today and *would have been* true at any time in the past—it is always true.

7. If *p* is always true, then it is impossible for *p* not to be true. [Modal Principle]

8. So, if it is true that there is a sea battle tomorrow, then it is *impossible* that there will *not* be a sea battle tomorrow.

9. If it is *impossible* that *not-p*, then *necessarily p.* [Possibility/ Necessity Exchange]

10. So, if there is a sea battle tomorrow, then it is necessary that there will be a sea battle tomorrow.

11. Therefore, Aristotle concludes, "Everything that will be, therefore, happens necessarily. So nothing will come about as chance has it or by chance; for if by chance, not of necessity" (*De Interpretatione* 18^{b}15-16).

12. And, "So [if bivalence holds] there would be no need to deliberate or take trouble (thinking if we do this, this will happen, but if we do not, it will not" (*De Interpretatione* 18^{b}31-32).

This version seems to make plain that Aristotle's modal principle does the brunt of the work and not bivalence or the law of excluded middle.

On the face of it the modal principle (7) may seem to endorse a more egregious modal fallacy than distributing necessity over disjunction. Aristotle's temporal understanding of necessity is certainly at odds with the contemporary Leibnizian conception. On the contemporary conception, whatever is necessary is true in all possible worlds. Suppose it has always been true that there is hydrogen in the universe. Does this make it necessary? It is easy to imagine a possible world in which there is no hydrogen. Though there has always been and always will be hydrogen in our universe, this fact is contingent, according to the Leibnizian understanding. Even if true future tense sentences have always been true—one would have always been speaking the truth if one had uttered such a sentence in the distant past—some future tense truths are nonetheless not true in all possible worlds and are therefore contingent. So, perhaps Aristotle's sea battle argument does no harm to the law of excluded middle or bivalence but instead defeats his own conception of necessity. The sea battle would then be a victory for the modern Leibnizian conception of modal logic.

That issue will surface later. For now consider what might motivate Aristotle's conception of modality. Before turning to the details, one should realize that Aristotle's conception is ontologically much more parsimonious than the modern proliferation of possible worlds. Aristotle construes modality within the bounds of the actual world. Those states of affairs that always hold *in the actual world* are necessary. Those that only hold sometimes are contingent. The resulting 'modal' logic will be extensional. Modal logic will simply involve first-order quantification over times. To say necessarily, *p* will simply be to say for all times *t*, *p* is true. If this conception of modality can be defended, then its ontological simplicity is a virtue not to be taken lightly. Aristotle's temporal conception of modality provides a natural explanation of the necessity of the

laws of logic and this virtue will to a large extent remove any oddity his conception suggests. Consider again the truth table for the law of excluded middle.

p	$\sim p$	p v $\sim p$
T	F	T
F	T	T

The table clearly shows that bivalence necessitates the law of excluded middle, the claim (B) above. But it also is obvious from the table that there will never be a time when it is false. It will always be true. So, Aristotle was right to say the law of excluded middle was itself necessary, the stronger claim (A) above. Moreover, this example illustrates how all tautologies, logical truths, will be necessary truths. Any tautology will have a truth table of Ts and so will *never* be false. All the truths of logic will be necessary by Aristotle's definition, and the explanation is intuitively appealing—they are *always* true. This account of modality remains nicely within the bounds of classical logic.

Although this explanation of the necessity of logical truths is attractive, one might think Aristotle's understanding of necessity threatens to turn all truths into necessary truths. But in *De Caelo*, he explicitly denies that all truths are necessary: "But there are also things possible and impossible, false and true absolutely. Now it is one thing to be absolutely false, and another thing to be absolutely impossible" (281^{b}6-9). He provides what anyone would call a standard and uncontroversial example of the distinction: "To say that you are standing when you are not standing is to assert a falsehood, but not an impossibility" (*De Caelo* 281^{b}9-10). So there are the ordinary contingencies. But this passage seems to contradict the passage in *De Interpretatione* in which he says whatever happens happens of necessity. This reading is a misunderstanding of Aristotle's temporal conception of modality however. Aristotle explains further: "A man has, it is true, the capacity at once of sitting and of standing, because when he possesses the one he also possesses the other; but it does not follow that he can *at the same time* sit and stand, *but at different times*" (*De Caelo* 281^{b}15-18, italics added). Because one *sometimes* will have the property of sitting, then it is possible for one to sit. Likewise, because one *sometimes* stands, then it is possible for one to stand. These explicate what is means for a person to have the capacity of standing and sitting. There are times when one does not have to be sitting and likewise for standing. But *at a given time*, if one is sitting, then it is impossible for one not to be sitting *at that time*. So, *at that time* it is necessary that the person is sitting. Aristotle thinks this conclusion can be generalized to any present particular fact. The same also holds for any past tense particular fact. If something has already happened at a given time, then it is impossible for the facts now to be otherwise. So, past particular facts are necessary. Aristotle's sea battle argument is to show that, based on the law of excluded middle, the same is true of future particular happenings. Since it was true to say yesterday that there will be a sea battle tomorrow (assuming there is sea battle) then it was a past particular fact that this was true. And if true in the

past, it is impossible for it to be false now to say there will be a sea battle tomorrow. Thus it is necessary.

Because his conception of modality falls within the bounds of classical logic, his conclusion seems to be a reductio of classical logic. Aristotle's solution seems to be to treat future tense statements differently from past tense and present tense ones. He says, ". . . statements are true according to how the actual things are . . . ." (*De Interpretatione* 19^{a}34). Many people interpret him to mean that since there are no actual future states of affairs, future tense statements are neither true nor false. In this case, the argument would suggest the need for a three-value logic. Other people think that the solution is to recognize unactualized possibilities. This case would suggest the need for a modal logic beyond the bounds of classical logic. Either way classical logic would be sunk.

Classical logic, however, can stay afloat on the sea of fatalism once we have recognized how weak Aristotle's sense of necessity is. Classical logic is only in trouble if one reads more into Aristotle's conclusion than is there. Call the case Aristotle makes for his conclusion "logical fatalism." *Logical* fatalism is true but harmless to free agency. A way to understand this is to realize that bivalence and the law of excluded middle amount to the claim: if *p*, then *p*. Of course, this truth is necessary—it is always true. If something happens in the future, then it would be true to say in the past that it will happen. If it is true in the past, then it is a past particular fact. The past cannot change. So, it is necessary (in that sense) that it will happen. But this is only to say that just as there is necessarily one past course of events, there will necessarily only be one future course of events. This conclusion is harmless. Past events, including free actions and decisions, may still play an important role in bringing about those future happenings. Aristotle's mistake is not found in a flaw in classical logic but in his concluding, "So there would be no need to deliberate or take trouble . . ." (*De Interpretatione* 18^{b}31). This *nihilistic* fatalism should be the only casualty of Aristotle's sea battle. Classical logic, with its innocuous *logical* fatalism in tow, has lived to fight another day.

NOTES

[1] W.V. Quine, "Two Dogmas of Empiricism," *From a Logical Point of View* (Harvard UP, 1953) 43.

[2] Aristotle, *De Interpretatione*, trans. J.L. Ackrill, *The Complete works of Aristotle* Vol. 1, edited by Jonathan Barnes (Princeton UP, 1984) 25-38. I will refer to the Bekker page numbers and cite them in the text after each quotation.

[3] Aristotle, *De Caelo* 281^{b}4-5, trans. J.L. Stocks, *The Complete works of Aristotle* Vol. 1, edited by Jonathan Barnes (Princeton UP, 1984) 447-511. I will refer to the Bekker page numbers and cite them in the text after each quotation.