SPINOZA, CANTOR, AND INFINITY

Lee Stauffer

The only consistency in the literature dealing with Spinoza's idea of infinity is that there is no consistency. Caird asserts that by 'infinite' Spinoza means, "removal of a limit," or "that which is unbound,"[1] while Wolfson asserts that "unbounded refers only to 'suo genere infinitum' and 'absolute Infinitum' means 'incomparable' in the sense of, 'not being included within a class of like things.'"[2] From this Wolfson infers that absolute infinity can not be diversified or divided,[3] while Harris asserts that, "blank and undifferentiated unity is not a whole and . . . can not be what Spinoza defines as substance."[4] Bennett,[5] following Joachin,[6] claims that 'infinite' should be understood in terms of Aristotelian logic as "All" while Wolfson would rather take it as similar to the complementarity of obversion.[7]

Considering this confusion, I would prefer to take Spinoza at his word and assume that when he uses the words 'infinitum' or 'infinité' he means what he says. If this results in some incoherence in his theory, it seems clear to me that this is a function of the general problems thinkers had with the notion of infinity during his time rather than a failure of modern readers to understand Spinoza. In taking this approach, I am supported by the extensive work done by Kline[8] showing that infinity (infinitum and less often infinité) are used as technical terms in Spinoza's Ethics.

Let us therefore consider the notion of infinity, both as it existed in Spinoza's time and as it is understood today. By the Renaissance 'infinity', except in common language, was no longer understood as the Greek did to mean a very large or uncountable number. There is evidence as early as the writings of Nicholas of Cusa[9] that 'infinity' had come to be recognized as a different sort of entity than the natural numbers. As Kline[10] shows, Spinoza's use of 'infinity' as 'very large' or 'uncountable' is limited to common language expressions meaning 'countless' and is never applied to 'attributum', 'ens', 'substantia', or 'Deus'. Thus, this meaning of 'uncountable' can not be the technical usage in the Ethics.

Another meaning which was common in the seventeenth century was that of 'unbounded' or 'unlimited'.[11] Unfortunately,

the mathematics of the time did not differentiate between these two quite different ideas. To be unlimited is not to have a limit (used here in the mathematical sense of 'end of a series'). Only ordered sets which express a sequence, such as the decreasing set of distances in Zeno's paradox of the arrow, can be said to have limits.[12] 'Unbounded', on the other hand, may be used to refer to entities, such as space, which are not sequential.[13] The physical universe is therefore said to be unbounded rather than unlimited.[14]

Neither of these ideas seems to me to be what Spinoza has in mind. First, as Wolfson[15] points out, Spinoza, following Maimonides, seems to wish to avoid seeing God as the kind of entity to which terms like 'limit' or 'boundary' could be applied (although Wolfson fails to distinguish between these two concepts). Spinoza, according to Wolfson, is not merely saying that God is unlimited or unbounded, but rather that the words can not be used with reference to God.

More importantly, neither idea is consistent with some aspect of Spinoza's system. 'Unbounded' when applied to Spinoza's system presents problems. Time is asserted by Spinoza to be infinite.[16] However, time is composed of an ordered, sequential set. Therefore, it can not be unbounded, only unlimited. But in the later part of the Scholium of Proposition 15, he compares substance to water,[17] which is certainly not sequential in nature. Since neither meaning will fit all of Spinoza's system, it seems clear that we must abandon this approach.

Rather, Spinoza would seem to be using the term in something like the mathematical sense which would be clarified by Bolzano, Dedekind and Cantor[18] into the modern notion of infinity as a part of set theory. There are two reasons which can be put forth in support of this interpretation. First, there is no other meaning available when 'uncountable,' 'unbounded' and 'unlimited' are excluded. Second, as we shall see, in the Ethics[19] he uses proofs similar to that which occasioned the development of set theory.

The mathematical notion of infinity, as it is used in set theory, can be developed, historically, in one of two ways, by division or by mapping.[20] The former approach is the one with which most philosophers are familiar, as it is the concept underlying Zeno's paradoxes. If follows from showing that between any two natural numbers, the sequence can be

infinitely divided. Based of Spinoza's explicit rejection of division of substance in Proposition 13[21], I do not think this is what he had in mind. Also, Zeno's paradoxes are resolvable by the calculus of infinitesimals in much the same vein as this argument of Spinoza's.

Rather, Spinoza seems to have in mind infinity as developed by mapping. Mapping consists in showing that two sets have the same cardinal number if for each member of one set there is a corresponding member of the other set. For example, if the set of philosophers in this room and the set of chairs in this room have the same (in this case finite) cardinal number, then each philosopher has a place to sit and each chair is occupied by a philosopher.

This approach to infinity produces what has been called "Galileo's Paradox", with which Spinoza was probably acquainted since it was well known to the mathematical and scientific community of the seventeenth century.[22]

Take the set of the real integers (one, two, three, etc.). Map on to this set the set of odd, real integers. Both are infinite. That is, you can keep adding another integer having a mapped, corresponding odd integer to the set for eternity. It appears that they have the same, i.e., infinite, cardinal number. But our intuition tells us that there should be more integers than odd integers, and indeed, that there should be twice as many. From this thinkers until the nineteenth century concluded that such a mapping was illegitimate for infinity since it was assumed that there could be only one infinity.[23]

When we examine Spinoza's Ethics, we find this sort of approach. In Proposition 13[24] he states, "Absolutely infinite substance is indivisible." The proof of this is as follows:

> If it were divisible, the parts into which it would be divided will either retain the nature of absolutely infinite substance, or not. In the first case, there would therefore be several substances of the same nature, which is absurd (Prop. 5). In the second case, absolutely infinite substance can cease to be, which is also absurd (Prop. 11).

and the following Scholium:

> The indivisibility of substance can be more easily understood merely from the fact that the nature of substance can be conceived only as infinite and that a part of substance can mean only finite substance, which involves an obvious contradiction (Prop. 8).

He further expands this in the Scholium of Proposition 15[25] in which he says:

> They (the examples) say that if corporeal substance is infinite, suppose it to be divided into two parts. Each of these parts will be either finite or infinite. If the former, then the infinite is made up of two finite parts, which is absurd. If the latter, then there is an infinite which is twice as great an another infinite, which is also absurd.

This is precisely the sort of problem presented by Galileo's Paradox.

I take this material to mean that Spinoza is asserting that if infinity were divided, the parts would not be infinite, either because there would be several substances of the same nature, i.e., infinite, which is not allowed by the definition of substance or infinite substance could be capable of ceasing to exist, thus being contingent, which is forbidden by Proposition 11, a form of the ontological proof. One could argue, and indeed philosophers[26] have argued, endlessly about whether the use of the ontological proof of substance as given by Spinoza is valid, thus attacking the second horn of the dilemma. However, I would rather grant Spinoza this principle and see if the other alternative, that of more than one infinite substance, is indeed absurd as Spinoza clearly thought.

Cantor, in the nineteenth century, in developing set theory, developed the diagonal proof, which asserts that there may be more than one order of infinity, that is cardinal number, for infinite sets, and indeed that there is an infinity of such sets. The diagonal proof consists of showing in a somewhat more elaborate way than Galileo's Paradox that unavoidably infinite sets of real numbers can be constructed. The advantage of his proof is that it combines both ways of constructing infinite sets.[27] The proof of the validity of this approach would take us too far afield. The importance to our consideration is simply that he denies the view of previous thinkers, such as Spinoza, that

orders of infinite are absurd. There is not one infinity, but many. We can put this in another way and assert that infinity behaves differently than finitude does with regard to wholes and parts.[28] However, we should consider the objections to Cantor's work which have been raised by mathematicians before applying it to Spinoza's philosophy.

First, it has been argued that this is true only in a theoretical sense and has no relevance to the real world. However, it seems to me that we can not allow Spinoza this way out as it would force him into a mathematical nominalist position,[29] which I feel would not be consistent with his views on mathematics in general. Although I am not sure that he was aware of the consequences, he makes so many of his points about metaphysical principles with geometric examples that it is hard to claim that he felt that mathematics was in some sense "not real." In particular, his objection to seeing corporal substance as not pertaining to God's essence because it is divisible[30] would seem to make his monism inconsistent with a nominalist position on this subject. Since a major aspect of Spinoza's system is that it must be taken as a whole or not at all,[31] I do not feel that an argument that mathematics isn't real or has nothing to do with the real world would be acceptable to Spinoza.

Alternatively, Cantor's work is sometimes attacked on the basis that ad absurdum proofs do not work.[32] Since Spinoza uses precisely this sort of proof repeatedly, he could not reject Cantor on these grounds.

Therefore it seems to me that Spinoza is incorrect in his statement of Proposition 13[33] that "Absolutely infinite substance is indivisible," and therefore that there can only be one infinite substance. This being the case, Spinoza's notion of substantial monism is seriously undermined by these later developments of mathematics.

One can only speculate how Spinoza, confronted with this problem, might have reacted. Certainly, in his own time, his argument on the nature of infinity was seen as unassailable mathematically. Even today, considerable resistance exists to Cantor's ideas, since they seem counterintuitive.[34] How Spinoza would have dealt with the problem of orders of infinity, had he known of it, I have no idea. However, I am convinced that, could we discuss the matter with Spinoza, he would find the work of Cantor a serious problem for his metaphysics.

NOTES

[1]John Caird, Spinoza (Philadelphia: Lippincott, 1988), p. 143.

[2]Harry Austryn Wolfson, The Philosophy of Spinoza (Cambridge, Mass.: Harvard University Press, 1934), pp. 133-134.

[3]Ibid.

[4]Errol E. Harris, "Finite and Infinite is Spinoza's System", in Speculum Spinozanum 1677-1977, ed. Siegfried Hessing (London: Routledge and Kegan Paul, 1977), p. 199.

[5]Jonathan Bennett, A Study of Spinoza's 'Ethics' (New York: Hackett Publishing Company, 1984), p. 76.

[6]H. H. Joachin, A Study of the Ethics of Spinoza (Oxford: Oxford University Press, 1901), p. 23.

[7]Wolfson, op. cit., p. 134.

[8]George L. Kline, "On the Infinity of Spinoza's Attributes," in Speculum Spinozanum 1677-1977, ed. Siegfried Hessing (London: Routledge and Kegan Paul, 1977), pp. 341-345.

[9]Armand A. Maurer, "Nicholas of Cusa" in The Encyclopedia of Philosophy, Vol. 5, ed. Paul Edwards (New York: Macmillan, 1967), p. 497.

[10]Kline, op. cit., p. 344.

[11]James Thomson, "Infinity in Mathematics and Logic," in The Encyclopedia of Philosophy, Vol. 4, ed. Paul Edwards (New York: Macmillan, 1967), pp. 183-184.

[12]Mary Tiles, The Philosophy of Set Theory: An Historical Introduction to Cantor's Paradise (Oxford: Blackwell, 1989), pp. 31-34.

[13]Thomson, op. cit., p. 184.

[14]Richard Morris, Time's Arrows: Scientific Attitudes Toward Time (New York: Simon and Schuster, 1984), p. 173.

[15]Wolfson, op. cit., p. 134.

[16]At least this is how Delahunty takes Spinoza's use of 'eternity'.
R. J. Delahunty, Spinoza (London: Routledge and Kegan Paul, 1985), pp. 120–121.

[17]Benedict de Spinoza, The Ethics (Ethica Ordine Geometrico Demonstrata) in The Rationalists, trans. R. H. M. Elwes (Garden City, New York: Dolphin Books, 1960), p. 192.

[18]Thomson, op. cit., pp. 184-185.

[19]Spinoza, op. cit., Prop. 13-15, pp. 188–193.

[20]Hans Hahn "Infinity," in The World of Mathematics, Vol. 3, ed. James R. Newman (New York: Simon and Schuster, 1956), pp. 1593-1611.

[21]Spinoza, op. cit., p. 188.

[22]Thomson, op. cit., p. 185.

[23]Ibid.

[24]Please note that this translation is taken from: Baruch Spinoza, The Ethics, in Classics of Western Philosophy, ed. Steven M. Cahn (Indianapolis: Hackett Publishing Co., 1977), p. 414. Other references are given from the complete source.

[25]Ibid., p. 415–416.

[26]Harris, op. cit., p. 200.

[27]William Kneale and Martha Kneale, The Development of Logic (Oxford: University Press, 1962), pp. 438-443.

[28]Hahn, op. cit., p. 1603.

[29]Ibid., pp. 1599–1600.

[30]Benedict Spinoza, op. cit., p. 190.

[31]Stuart Hampshire, Spinoza (London: Faber and Faber, N.D.), pp. 42–43.

[32]Hahn, op. cit., pp. 1599–1600.

[33]Benedict Spinoza, op. cit., p. 188.

[34]Hahn, op. cit., p. 1605.