

RUSSELL'S CONTRADICTION AND THE PROLETARIAN PARADOX

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A sardonic twist in paradox is afforded by comparing the paradox in Chapter 10 of Russell's Principles of Mathematics with Lukács' History and Class Consciousness analysis of the role that Lenin assigned to the proletariat in revolution and government. Russell's paradox was a useful spur to mathematicians to rework the axiomatic foundations of their science and to be careful in defining such innocent terms as 'all', 'class', etc. Lukács' paradox is also useful in so far as it gives a bitter warning that the proletariat as a 'class' is a most unlikely candidate for leading the 'proletarian revolution' or for ruling when the proletarian dictatorship is established.

My original intention was to inquire if certain general social ruptures might have produced the paradoxes by creating ruptures in the smooth flow of "collective" thought, but I decided that in a brief paper it would be enough to list a few of the special circumstances confronting Russell and Lenin, when the one discovered the logical contradiction and the other laid his fateful long-range plans for revolution.

Russell's paradox arose directly from his studies of paradoxes which had been discovered in the field of transfinite numbers. His own contradiction concerns the class of all classes that are not self-members: is this class a member of itself? To answer this question, let us first represent Russell's class \underline{r} , and create a formula showing that any class \underline{x} can be a member of ("memb") \underline{r} if, and only if, \underline{x} is not a member of ("memb") itself:

(1)

$$(\underline{x}) (\underline{x} \text{ memb } \underline{r}) \leftrightarrow \neg(\underline{x} \text{ memb } \underline{x}).$$

Then whether or not \underline{r} is a member of itself can be answered by substituting \underline{r} for \underline{x} in (1):

(2)

$$(\underline{r} \text{ memb } \underline{r}) \leftrightarrow \neg(\underline{r} \text{ memb } \underline{r}),$$

from which the explicit contradiction can be derived:

(3)

$$(\underline{r} \text{ memb } \underline{r}) \ \& \ \neg(\underline{r} \text{ memb } \underline{r}).$$

When Russell informed the logico-mathematical community of his discovery there was somewhat of a flurry. Frege, e.g., is reported to have been in doubt about the validity of his own attempts to construct a logical foundation for arithmetic; but recuperative moves were immediately undertaken by all concerned. Russell developed his theory of types and others began to repair the axiomatic substructure. The immediate problem was pinpointed by

Russell as being our ordinary common sense. Concern was expressed over what logical rules, if any, had been violated by Russell's paradox and the numerous contradictions discovered by others. J. Van Heijenoort in his article, "Logical Paradoxes" in the Encyclopedia of Philosophy (1967) offers this assessment:

There is no one problem of the paradoxes. The problems are of different types. They are not due to some infraction of one specific law of logic ('vicious circle'), nor are they simply mistakes to be removed by some ad hoc corrective. The paradoxes actually reveal conflicts in our logical intuitions. Following a logical path, we reach a conclusion; following another path. . . we reach a contrary conclusion.

Although these remarks refer to a number of paradoxes, they reflect some of the mystique that has been associated with the Russell contradiction. Russell himself thought that he would be able to resolve the problem a few days after its discovery, but the more he thought about it the slipperier and more significant it became to him. From our point of view, however, we need but remember that \mathfrak{r} is based on conflicting ideas because this is the way Russell defined it. Nor do we need ad hoc constructions to uncover the major logical error concealed within it, for it is easy to prove \mathfrak{r} violates the "class theoretical law of identity" when that law is construed as being the principle that each class includes itself. For example, a class K is included in another class L if and only if the members of K are also members of L. Appropriately abbreviated, the formula is

(4)

$(K \text{ incl in } L) \leftrightarrow (\mathfrak{x})(\mathfrak{x} \text{ memb } K \rightarrow \mathfrak{x} \text{ memb } L).$

Therefore for \mathfrak{r} to be included itself it must withstand the following formula:

(5)

$(\mathfrak{r} \text{ incl in } \mathfrak{r}) \leftrightarrow (\mathfrak{r} \text{ memb } \mathfrak{r} \rightarrow \mathfrak{r} \text{ memb } \mathfrak{r}),$

which it cannot do since the right member of (5) is denied by (3) above. Therefore it is false that \mathfrak{r} is included in \mathfrak{r} . The Russell class is thus reduced to being less than the null class since this latter class at least has the dignity of including itself.

Russell's paradox must suggest to those acquainted with Hegel's Logic that it resembles the first "movement" of the dialectic, and something of this nature possibly occurred to Russell, for in the same context in which he attributed the contradiction to common sense, he offered this pre-philosophical grounding as evidence that it presupposes no philosophy, certainly not Hegel's (which "nourishes itself on contradiction.") But what Russell

attributed to common sense is not dissimilar to what Hegel ascribed to the understanding, although the attitude of the one toward contradiction certainly differed from that of the other. What Van Heijenoort says about the conflicts in intuition is also paralleled by Hegel's account of Kant's antinomies: Kant corrected the old metaphysics which issued statements without regard to their opposites; consequently he (Kant) "tried to prove that the. . . statements could be met by other statements of contrary import with equal warrant and equal necessity."¹

Oddly enough, Russell's discovery of the paradox was preceded by a period of profound despair in which opposing elements are discernible. This indisposition began in the sick room of Mrs. Whitehead in February of 1901 and lasted intermittently for several years. While witnessing the poor woman's suffering, he was suddenly aware of the stark aloneness of each soul; yet as he himself suffered anguish, he found himself uplifted with a sense of triumph that he could both suffer and master the pain, using it as a gateway to greater wisdom. This is why he could write in A Free Man's Worship (which was begun at that time) that it is "only on the firm foundation of unyielding despair" that "the soul's habitation can henceforth be safely built." In May of the same year he discovered the paradox, which he described in his Autobiography as "an intellectual set-back almost as severe as the emotional set-back I had had in February." In the years that followed, the despair gradually lifted and the intellectual turmoil was virtually ended by 1906 when his theory of types was perfected.

During the same years that Russell struggled with his personal contradictions, V.I. Ulyanov--Lenin--went through the most decisive period of his life. In early 1900 he returned from a three year Siberian exile with the conviction that the Russian Social-Democratic Party could lead the masses into a successful revolution only if it were thoroughly reconstructed. As things stood organization was lax and respect for Marxist theory indifferent. Members of his restructured Party would be theoreticians, saboteurs, propagandists, and agitators. Above all, they would be disciplined to take orders from the Central Committee. To further this aim, it would be necessary to institute an all-Russian newspaper to issue orders and to discuss Party issues. Divergent points of view would be allowed so long as it was possible for editorial correction to set the record straight. Accordingly, Iskra was established within the first year of his return from exile, and Lenin remained as editor of this publication until the Bolshevik-Menshevik split in 1903. Not surprisingly, this split arose during a "unification" congress which was held to iron out differences, one of these differences being over the centralization of power.

Lenin outlined his organization plan in Iskra articles and in What Is to Be Done? which first appeared in Stuttgart in March, 1902. Of especial concern to this discussion is his opinion about the leadership role of the proletariat in the forth-coming struggle against tsarism and capitalism. Since he so strongly endorsed centralism, the die was cast against leaving the conduct of the revolution to the workers. The working class by itself, he said, "is able to develop only trade union consciousness. . . the conviction that it is necessary to combine in unions, fight the employers and strive to compel the government to pass necessary labour legislation, etc."² The theory of socialism, Lenin added, was developed by representatives of the "propertied classes, the intellectuals." Marx and Engels were bourgeois intellectuals. A talented worker, of course, might make the grade, but not as a laborer. The vaunted vanguard of the proletariat must be staffed by intellectuals, as a revolution is unthinkable without theory. Moreover, the vanguard itself must be constantly policed and purged in order to maintain correctness of theory and flexibility of action.

This stringency of control was as necessary after the revolution as it had been long before the struggle began. In a pamphlet issued in January, 1921 which was addressed to certain mistakes of "Comrades Trotsky and Bukharin" Lenin thought it expedient to issue the following reminder and threat:³

The state is a sphere of coercion. It would be madness to renounce coercion, especially in the epoch of the dictatorship of the proletariat, so that the administrative approach and 'steerage' are indispensable. The Party is the leader, the vanguard of the proletariat, which rules directly. It is not coercion but expulsion from the Party that is the specific means of influence and the means of purging and steeling the vanguard.

Thus the vanguard--the Party--rules. A little later he re-emphasized this point by referring to the "Central Committee and the whole Party" as the ones who ran things or governed the country. The proletariat therefore had fought in a revolution that it had not led and was the subject of a dictatorship that bore its name.

This was perhaps the major problem confronting Georg Lukács in History and Class Consciousness. But how could the proletariat lead the proletarian revolution if as a class it could not attain authentic proletarian consciousness? Lenin had said that the workers when left to themselves cannot advance beyond trade union consciousness, i.e., they do not comprehend any moves beyond bargaining for crumbs from the capitalist table. This is a

kind of bourgeois thinking. Hence Lukács could confidently assert that "we must never overlook the distance that separates the consciousness of even the most revolutionary worker from the authentic class consciousness of the proletariat." But even this proletarian backwardness can be explained by the "Marxist theory of class struggle and class consciousness." For:⁴

The proletariat only perfects itself by annihilating and transcending itself, by creating the classless society through the successful conclusion of its own class struggle. The struggle for this society. . . is not just a battle waged against. . . the bourgeoisie. It is equally the struggle of the proletariat against itself: against the devastating and degrading effects of the capitalist system upon its class consciousness.

The conditions laid down here specify that any class x that realizes the full potential of being the proletariat--it bears the authentic proletarian consciousness by being able to think like Marx and Lenin--succeeds in annihilating itself by vanishing into the classless society that it has helped to build. When stylized, Lukacs' statement assumes the familiar form:

(6)

$(x)[x=P \quad \neg(x=P)]$

It is evident that in (6) we have the representation of the dialectical justification of the role played by the proletariat in Lenin's revolution and dictatorship, and at the same time the formula contains a judgement of the capacity of the proletariat to perform in any Marxist revolution or dictatorship. But what is logically depicted at (6) is an r-type class.

The temporal juncture of the logical and social paradoxes can only be noted but not explained here. At a simple level we can say that the times seemed to call for both men to undertake enormously difficult tasks. Russell's search for mathematical foundations resulted in Principia Mathematica, and Lenin's revolution within a revolution was the beginning of the transformation of a vast, polyglot, and largely agrarian society into a technocracy. Sputnik has taught us about the reality of the dream. Russell's paradox came from thinking about attempts of others to impose the concept of class, which is generally unrelated to the concept of infinity by common sense, upon the concept of infinity. Lenin, on the other hand, needed the propaganda edge provided by liberal use of such Marxist phrases as "proletarian revolution" and "dictatorship of the proletariat" while recruiting a band of professionals really capable of leading and ruling. Finally we might note that Lukács was trying to show (as other Marxists have done in other circumstances)

that Lenin was not resorting to lies or subterfuge in dealing with the proletariat as certain prominent Marxists accused him of doing; rather he was skillfully using dialectics.

NOTES

1. The Logic of Hegel, trans. William Wallace, 2nd. ed., rev. (Oxford: Oxford University Press, 1959), p. 100.
2. V.I. Lenin, What Is To Be Done? (Peking: Foreign Languages Press, 1975), p. 37.
3. V.I. Lenin, On the Question of Dialectics: A Collection (Moscow: Progress Publishers, 1980), p. 97.
4. Georg Lukács, History and Class Consciousness, trans. Rodney Livingstone (Cambridge, MA: M.I.T. Press, 1971), p. 80.