PEIRCE, BEDDOES, AND PRAGMATICISTIC ABSTRACTION: AN INTRODUCTION[1]

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In his proposed 1907 article for *The Nation* entitled "Pragmatism," Charles Peirce gave an explanation of his general system with specific regard for the importance of the eighteenth-century English physician and mathematician Thomas Beddoes, M.D. (1760-1808):

> The method of pragmatism is simply the experimental method, which (taking the word "experiment" in its widest sense, so as to make it applicable to cases in which the fulfillment of the conditions has to be waited for instead of being artificially produced) is the invariable procedure of all successful science. Thomas Beddoes showed, as early as 1792, that it is the procedure even of mathematics. (Peirce 1907/*MS* 320: 29)

Three important questions arise from these remarks: what is this experimental procedure of mathematics that Peirce credits to Beddoes, what expression does this procedure or method take for Beddoes, and how might Beddoes' method compare with Peirce's system in general and with Peirce's ideas on experimentalism, diagrammatic thought, and abstraction individually? My present introductory account, then, has two functions: (1) to address these questions by way of providing an initial investigation of Beddoes and his observations on mathematics as important historical antecedents of and influences on Peirce's pragmaticism, and (2) in doing so, to bring to your attention, probably for the first time, a number of textual fragments by Beddoes that reveal him to have been a pragmaticist in all but name.

A sketch of Beddoes' life includes the following items. He was born in Shifnal, Shropshire, and received his doctor of medicine degree from Oxford in 1786. Immediately thereafter he became acquainted with Lavoisier in Paris, and he returned to Oxford in 1788 as a reader in chemistry. Standard sources remark that he attracted audiences for these chemistry lectures larger than any that had been seen at Oxford since the thirteenth century (see Gillispie 1970, 56; Stephen and Lee 1917, 94). Frictions created by his sympathies with the current French Revolution, however, contributed to his resignation of this lectureship in 1792. His *History of Isaac Jenkins*, an essay on the evils of intemperance, was brought out the following year and, having reportedly sold around forty thousand copies, must have been something of a best seller by standards of the day. The list of publications supplied in John Edmond Stock's 1811 *Memoirs of the Life of Thomas Beddoes, M.D.*, which omits a number of contributions Beddoes made to various journals of the day, nevertheless cites forty-five titles covering subjects as diverse as medicine, natural history, politics, librarianship, and philosophy. His 1799 *Essay on Consumption* was admired by Kant.

In 1798, with technical and financial assistance from James Watt and Josiah Wedgewood, Beddoes established at Clifton his Pneumatic Institution, a kind of hospital whose aim was to investigate the treatment of disease by inhalation of and confined exposure to various gases. Nitrous oxide seems to have been a favored medicinal in these high-minded experiments. Notable was Beddoes' selection of Humphrey Davy, then aged nineteen, as the Pneumatic Institution's first superintendent. Beddoes continued his association with this Institution until the year before his death. On that occasion Coleridge wrote, "I felt that more had been taken out of my life by this than by any former event." Davy said of him that:

> He was reserved in manner and almost dry. Nothing could be a stronger contrast to his apparent coldness in discussion than his wild and active imagination, which was as poetical as Darwin's. He had talents which would have raised him to the pinacle of philosophical eminence, if they had been applied with discretion. (Stephen and Lee 1917, 95)

Beddoes' work noted above by Peirce was his *Observations on the Nature of Demonstrative Evidence* of 1792. There Beddoes took great care to illustrate his system of demonstration to be based on and composed of mathematical experimentalism:

> On examining a train of mathematical reasoning, we shall find, that at every step we proceed upon the evidence of the senses; or, to express myself in different terms, I hope to be able to shew *that the mathematical sciences are sciences of experiment and observation, founded solely upon the induction of particular facts, as much so as mechanics, astronomy, optics or chemistry*. (Beddoes 1792, 15)

In particular, Beddoes was interested in what he took to be certain experimental implications in Euclid's *Elements*, their consequences, and how these items could be put to use both in examining Euclid's system itself and in the education of students of geometry. Shortly we'll delve into some technical aspects of these two points.

Beddoes was concerned, in his elaboration of mathematical experimentalism, to respond to the nonexperimental, rote or mechanicalistic method of teaching, learning, and even performing demonstrations or proofs in geometry that seems to have been pervasive in his day. Particularly, in the Dedication to *Observations*, he took exception to this doctrine as espoused by James Harris (1709-80) in *Hermes* (1771). Among his dedicatory remarks addressed to his colleague and friend Davies Giddy, Beddoes quoted the most offending section of Harris' *Philosophical Inquiry* (as *Hermes* was subtitled). Beddoes wrote to Giddy:

> Fortunately for the diffusion of just sentiments, Mr. Harris has lost that authority

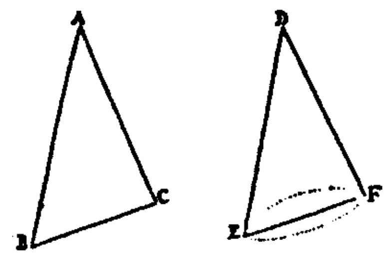
> which even among the learned he maintained too long. Our young men, however, I believe, still frequently resort to *Hermes* for that instruction, which he has not to supply. For observe, I beseech you, what they will learn from this once redoubted doctor of *universals*, concerning mathematical reasoning.— "It is somewhat remarkable," says he, sarcastically glancing at the attention paid to the physical sciences, "amid the prevalence of such notions, that there should still remain two sciences in fashion, and these having their certainty of all the least controverted, *which are not in the minutest article depending upon experiment*. By these I mean ARITHMETIC and GEOMETRY." In a curious note, but which is too long to be inserted entire, he has the insolence to subjoin, "I would not be understood, in what I have here said, or may have said elsewhere, to undervalue EXPERIMENT; whose importance and utility I freely acknowledge, in the many curious nostrums and choice receipts, with which it has enriched the necessary arts of life. Nay, I go farther—I hold *all justifiable practice in every kind of subject* to be founded in EXPERIENCE, which is no more than the result of *many repeated* EXPERIEMENTS. . . . In the mean time, while EXPERIMENT is thus necessary to all PRACTICAL WISDOM, with respect to all PURE and SPECULATIVE SCIENCE, it has not the least to do. For who ever heard of *Logic*, or *Geometry*, or *Arithmetic* being proved *experimentally?*" (Beddoes 1792, v-vii, quoting Harris 1771, 351-53)

Beddoes responded to Harris's convictions with an example from Euclid, viewing it not as an item of "pure and speculative science" completely divorced from all experimental procedures, an item to be verified or proved solely within the axiomatized system of which it is a part and then recalled by

rote, but as an item of knowledge to be learned and understood by actual experimental observation. The nature of this illustration is such that, I think, even persons such as myself, who have little formal training in mathematics past trigonometry, can clearly observe in it a basic aspect of the experimental nature of mathematics for which Beddoes and Peirce both argued.

Now, using the diagram supplied below and your own poetical imagination, consider Beddoes' example. His example was the fourth proposition of the first book of Euclid's *Elements*, a selection which, with its accompanying axiom, "as it is called, . . . may be regarded as the corner-stone of geometrical reasoning" (Beddoes 1792, 20). This proposition states that, given two scalene triangles with pairs of respective sides and the interior angle produced by these sides being equal to one another, the remaining respective single sides and the two new interior angles created by these sides will also be equal, and finally the two completed respective triangles will also be equal to one another. The axiom involved is concerned with coincidence and equality, given as "Things which coincide with one another are equal to one another" (Heath 1956, and Hutchins 1952, 2,4). It is important to note that Beddoes considered this axiom as well as the proposition in question both to be, when rightly considered, results of experimental observation rather than examples of Harris' pure and speculative science.

To commence his own demonstration, then, Beddoes supplied a diagram of the two triangles involved:



Triangles of Euclid's Fourth Proposition
(Beddoes 1792, 20)

Beddoes' point was that instead of merely accepting Harris' mechanicalistic claim that this kind of proposition could only be demonstrated in accordance with and within the closed system of "pure and speculative science" of which it is a part, an actual experimental alternative could be given. In doing so, Beddoes seems to enunciate something very like Peirce's distinction between dyadic, mechanical actions and triadic, intelligent, or creative relations. Thus, he also seems to have in mind something similar to Peirce's conception of self-control. Beddoes wrote to Giddy:

> The more I consider the subject, the more I am inclined, in spite of Mr. Harris, to believe not only in the possibility, but the utility of rendering the elements of geometry *palpable*. (Beddoes 1792, vii, emphasis added)

Actual "models," involving "the use of [the student's] hands and eyes," Beddoes claimed, "would make the study infinitely more engaging" (Beddoes 1792, vii, 19). Supplied with the appropriate equipment, then, he describes how the experimenter may proceed:

> If you have a *model* of each triangle cut out of pasteboard, or any other material, you are to place the point A upon D, which is to commence an experiment; if the triangles be only traced upon a surface, you are to *imagine* A placed upon D, which is to *imagine* the commencement of an experiement. (Beddoes 1792, 21, emphasis added)

An important implication of this description had already been made clear, namely, that "These experiments may, indeed, be called *mental* experiments, since the appeal is made to recollection, and they are commonly repeated in thought" (Beddoes 1792, 18-19). Employing in this particular instance, then, a combination of mental and physical models or diagrams, the observer uses these models experimentally to

compare respective sides and angles, actually *seeing* what coincidence and equality in length and angle mean in such an instance, and finally actually observing, in both physical approximations and abstract mental versions, a coincidence of the two triangles. Beddoes especially emphasized the value of such experimentation at the moment when the final two respective sides are compared.

> Then BC must fall upon EF; "and why?"—make the experiment; you cannot place two straight sticks, or trace two straight lines, so that [they] encompass a space, try as long as you please and satisfy yourself. . . . I have been purposely prolix in this demonstration, to shew how it begins in experiment, goes on by experiment, and ends in an experimental conclusion. (Beddoes 1792, 23, 24)

Additionally, from his discussion of the fifth proposition from the first book of Euclid's *Elements*, which is concerned with properties of isosceles triangles, and how the demonstration of this proposition relies on a working out of the fourth, he made clear that such "experimental conclusions" can make for the beginnings of new experimental observations (or, we might note, Peircean abstractions):

> The fifth proposition is said to have stopped many students of geometry in their career; this is owing partly to the length of the demonstration, and partly to the complication of the diagram. The demonstration is, however, nothing but the result of the experiments in that of the fourth combined with the results of two other very simple experiments. . . . In this manner does every demonstration proceed upon the results of experiments, as the reader will find, in as many instances as he shall take the pains to examine. And since the appeal in demonstrative reasoning is

> always made to what is now exhibited to the senses, or to what we have before learned by the exercise of the senses, too much pains cannot be taken, at the commencement of the study of geometry, to satisfy the mind of the learner by appealing to his senses. The more distinct and deep the impressions of sense are at the beginning, the greater will the power of *abstraction* afterwards be. . . . We *abstract*, when we narrow the sphere of sensations and dwell upon impressions, or when we recollect the ideas thus acquired . . . or learn to reconsider each separate perception, as well as to combine them anew. (Beddoes 1792, 25-26, 27-28, 29)

To summarize and paraphrase Beddoes' response to Harris: the mathematical sciences, as demonstrated, *are* indeed sciences of experiment and observation and not exclusively examples of "pure and speculative science." Models or diagrams, whether mental or actual "detached figures," as I term them, are used by geometers in their abstractive investigations, to draw experimentally observable consequences. These consequences or abstractions may then form the basis for further experimental investigations.

So, how in fact do certain aspects of Peirce's system compare with Beddoes' ideas of mathematics being an observational, experimental science, diagrammatic thought being required in such mathematics and this requirement itself being demonstrated by such mathematics, and the abstractive processes and results of such diagrammatic experiments being susceptible of yet further experiments and observations?

To begin addressing these questions, let us review briefly where Peirce located mathematics within his overall system:

MATHEMATICS
PHILOSOPHY
  Phenomenology
  Normative Science
    Esthetics

    Ethics
    Semiotic [Logic in the general sense]
      Speculative Grammar
      Critic [Logic in the narrow sense]
      Methodeutic
SPECIAL SCIENCES [Physics, Psychics, etc.]
(Adapted from Ketner 1983, 336)

Peirce constructed this Classification of the Sciences according to "the general principle set forth by Auguste Comte, that is, in the order of abstractness of their objects, so that each science may largely rest for its principles upon those above it in the scale while drawing its data in part from those below it" (Peirce 1898/*MS* 437, 20) Mathematics, then, was naturally placed at the head of this classification, as mathematics for Peirce was concerned with the study of pure hypotheses and so should influence all those classifications beneath it. In a letter to Francis Russell in 1894, for instance, he disclosed that he considered it his "special business to bring mathematical exactitude, modern mathematical exactitude into philosophy, and to apply the ideas of mathematics in philosophy" (Peirce 1894). He had made clear the relation between mathematics as this most abstract of classifications and mathematics as nevertheless an observational, experimental science already by 1886 in his *Nation* review of Francis Ellingwood Abbot's *Scientific Theism:*

> Of all the sciences—at least of those whose reality no one disputes—mathematics is the one which deals with relations in their abstractest form; and it never deals with them except as embodied in a diagram or construction, geometrical or algebraical. The mathematical study of a construction consists in experimenting with it; after a number of such experiments, their separate results suddenly become united in one rule, and our immediate consciousness of this rule is our

> discernment of the relation. It is a strong secondary sensation, like the sense of beauty. (*N* I: 73)

Thus, considering again Beddoes' experimental investigation of Euclid's proposition about scalene triangles, we see that Peirce's classificational scheme holds even within mathematics itself: the "data" of Beddoes' geometrical diagram or construction are drawn upon to foster our discernment of those abstract relations of equality in length and angle found higher in this "scale." Peirce reinforced this point about the value of diagrammatic thought for reasoning in a letter to J. M. Hantz the following year, while echoing Beddoes' use of "hands and eyes" in geometrical research:

> For my part, I hold that reasoning is the observation of relations, mainly by means of diagrams and the like. It is a living process. This is the point of view from which I am conducting my instruction in the art of reasoning. I find out and correct all the pupil's bad habits in thinking: I teach him that reasoning is not done by the unaided brain [alone], but needs the cooperation of the eyes and hands. Reasoning, as I make him see, is a kind of experimentation, in which, instead of relying on the intelligible laws of outward nature [alone] to bring out the result, we depend upon the equally hidden laws of inward association. I initiate him into the art of this experimentation. I familiarize him with the use of all kinds of diagrams and devices for aiding the imagination. (Peirce 1887)

And in the sixth lecture from his 1903 Harvard series "Pragmatism as a Principle and Method of Right Thinking," given in Sever Hall, Peirce described the functions of abstraction in a manner strikingly similar, I find, to Beddoes' use of this term:

> We not only have to select the features of the diagram which it will be pertinent to pay attention to, but it is also of great importance to return again and again to certain features. Otherwise, although our conclusions may be correct, they will not be the particular conclusions at which we are aiming. But the greatest point of art consists in the introduction of suitable *abstractions*. By this I mean such a transformation of our diagrams that characters of one diagram may appear in another as things. (*CP* 5.162)

So it seems quite clear that Peirce and Beddoes were indeed of like mind on the issue of mathematical reasoning being an observational, experimental science relying explicitly on the use of diagrams or models, whether such diagrams be constructed in "the imagination" or as "detached figures." It seems, in fact, that the competent investigator utilizes both.[2] Equally strong, I think, is an agreement of Peircean and Beddoean notions that such diagrammatic or abstractive reasonings could and should, in their turn form abstractions with which to carry forward further experiments. I think Beddoes would have agreed completely with Peirce that such abstractive reasonings require triadic relations for their successful operation, for, as they both might have asked, "How *could* a Harris-style dyadicism foster true abstraction?"

I will close with a foreshadowing of further research. In taking abstraction in both a narrowing or selective sense and also in a generalizing or transformative sense, Beddoes understood already something closely akin to Peirce's distinction of prescissive and hypostatic or subjectal forms of abstraction and perhaps also the method of pramaticism itself (see *NEM* III: 917; *CP* 5.449). By means of this and other observations, our understanding of Beddoes as an important and perhaps major historical influence on Peirce and pragmaticism can be advanced.

NOTES

[1]My gratitude goes to the President and Organizing Committee of the Charles Sanders Peirce Sesquicentennial International Congress, held at Harvard University, who provided an opportunity for me to present an expanded account of this historical and methodical connection between Beddoes and Peirce during opening sessions at Sever Hall, Cambridge, on 6 September 1989.

[2]It is quite interesting to review the extracts here from Beddoes and Peirce in light of Peirce's claim that diagrammatic thought is not limited to optical or visual examples, but is likewise exercised in acoustic or auditory and tactile ways (see *CP* 3.418; *NEM* III: 869-80). Peirce further connected all three of these varieties of diagrammatic thought with both corollarial and theorematic forms of deduction. As a classical pianist of some experience, I find it especially interesting, given all this, to continue investigations into how acoustical, tactile, and optical components of, say, Rachmaninoff's last *étude tableaux* (op. 39, no. 9) combine to produce theorematic forms of reasoning; that is, aren't such pianistic endeavors pragmaticistic?

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