Incompatibility and a Logic of Rejected Concepts

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Natural language is neither pure nor innocent. It contains concepts that have inferential liaisons preventing us from applying those concepts directly to anything in the world. Think, for example, of a concept from a discarded scientific theory, such as that of phlogiston. Someone might reasonably deny that *anything* contains phlogiston. However, if instead we were to say that nothing has phlogiston, then by the very meaning of the term (namely that phlogiston is necessary to support combustion), that would seem to imply that *nothing* is combustible. Thus, one might wish to deny of anything that it either has or doesn't have phlogiston. My goal here is to introduce a new kind of propositional content, formally symbolized by striking through a candidate atomic formula, which is meant to capture, at least in part, the rejection of the actual application of some portion of the subsentential conceptual content of that formula. Oscar Wilde's prosecutor is said to have posed the following charge, "I put it to you, Mr. Wilde, that this is *blasphemy*. Is it or is it not?" To elide an obvious trap set up for him, Oscar Wilde is said to have replied, "Sir, 'blasphemy' is not one of my words." My project is to provide an initial step towards a formal understanding of the logic of attitudes like that of Mr. Wilde's toward blasphemy.

So on the syntactic front, let us stipulate that for any atomic proposition P, $\not{\mathrm{P}}$ is a well formed formula (wff). Bear in mind that I'm not conceiving the strike-through as a general sentential operation. I mean to apply it here only to atomic formulas (or sub-sentential components), not logically compound contents, and it is not meant to be iterated.

The strikethrough is meant to symbolize the rejection of some of the conceptual content contained in that atomic formula. As such, it signifies in a given context of discourse the preclusion of any direct application of the rejected concept or term involved – be that application affirmative or negative. So the rejection of P, symbolized by ~~P~~, is to be understood as incompatible with both P and also with ~P.

Such preclusion might seem to indicate on my part some rejection of the law of excluded middle, and so the adoption of a logic that introduces truth-value gaps, or at least some truth-value beyond the standard true and false. The latter inference is *not* my intent! Instead of introducing new semantic values or gaps, I'm actually trying to introduce new kinds of assertible *contents* (or things that can legitimately be said), which cannot be handled in truth-functional semantics except through such drastic measures. Nevertheless, there is something to the suggestion that I'm advocating a non-standard logic, for I'm not at all enamored by semantic theories centered around the specifications of truth-values.[1] Instead, my preferred semantic framework is that of Brandom's incompatibility semantics, in which the semantic value of a propositional content is understood to be the set of sets of all other such contents commitment to which *precludes* commitment to it (Brandom, *Between Saying and Doing* 117-175). So one lesson that could be taken away from this paper concerns the advantages of adopting a semantics that takes incompatibility relations to be its primitive over more traditional truth-functional alternatives, insofar as that enables us to overcome the challenges of dealing with propositions we neither wish to assert nor deny, without having to do anything so drastic as inventing whole new semantic values or gaps.

Accordingly, the semantic primitive in such an incompatibility framework is that of an *incompatibility frame*, from which one can discern all sets of materially incompatible commitments. The trick then is to show how incompatibility sets for logically compound formulas can be generated recursively out of such an incompatibility framework. In his Locke lectures, Brandom has shown how this may be done with negation and the familiar sentential operators, as well as modal operators, which behave just like the box and diamond in S5.[2]

Brandom introduces ~P as a content which is "minimally" incompatible with P. That is, anything incompatible to P (in particular, those sets of sentences which are stipulated to be materially incompatible with P by an incompatibility frame) will entail ~P.[3] And so, anything incompatible with ~P will be in the *intersection* of the sets of sets of sentences that are incompatible with sentences incompatible with P. To be sure, that's a mouthful! The basic idea is that the negation of P is supposed to capture what is in common with everything incompatible with P – namely they all rule out P.

One thing to notice here is that Brandom's approach is one of introducing new assertible contents (analogous, say, to the introduction of the negatives to the

natural numbers), which may or may not not line up with (or be logically equivalent to) anything that otherwise could already be said without the use of such contents. Brandom's project, which goes under the heading of "logical expressivism," is to domesticate these introduced contents by specifing the inferential roles of negations and other logical compounds, and it happens to be a virtue that these compounds can be shown to be inferentially conservative. Given the familiar rules for introducing and eliminating negations, for instance, negation so understood will not license any new inferences between non-negation-involving contents that weren't already materially licensed before by an incompatibility frame.[4]

In a similar spirit, I wish to extend the domain of possible assertible contents by introducing new contents, grammatically marked with the strikethrough, which we may understand to represent an even "deeper" level of incompatibility than the negation. The task is to specify the inferential roles of strikethrough contents by saying in turn what those strikethrough contents are incompatible with. On Brandom's understanding, ~P is understood to be entailed by anything incompatible with P. The introduction of P̶, however, forces a slight revision of that idea. Instead, any set of commitments incompatible to P will either have to *entail* ~P or it will have to *include* P̶. Moreover, the strikethrough content P̶ is further stipulated to be incompatible with ~P. The effect of these stipulations is straightforward: whereas ~P is still straightforwardly incompatible with anything entailing P, P̶ is understood to be incompatible with anything entailing *either* P or ~P. In that precise sense, P̶ is more *deeply incompatible* to P than is ~P.

Strikethrough contents may in turn embed to form logically compound contents, which behave in the familiar, classical fashion. The conjunction of P̶ with some formula Φ will entail each in turn, and be jointly entailed by both. And their disjunction will entail one or the other, and be precluded by the preclusion of either.[5] Of perhaps more interest is the negation of some strikethrough content. Since ~P̶ is stipulated to be minimally incompatible with P̶, anything incompatible with P̶, including most importantly P and ~P, will in turn entail ~P̶.

With the advent of strikethrough contents, we can see that in a context in which one is coherently committed to P̶, one can thus coherently be *precluded* from commitment to (P v ~P). Other than stubborn allegiance to the necessity of the law of excluded middle, perhaps out of some misguided fetishization of truth-functionality, I see no principled objections to the coherent introduction of such new contents and contexts. Indeed, those very semantics seem designed precisely to cover up the deeper level of incompatibility revealed by strikethrough contents.

The last three paragraphs, by the way, illustrate another important theme running through Brandom's work: "expressive bootstrapping." The conceptual resources sufficient to describe the proper use of some expressive device need not require one actually to deploy that device. Thus, one can begin with relatively meager expressive resources and show how to say something important and useful

that could not otherwise be said in the limited vocabulary.[6] The introduction of strikethrough content allows us to say something that otherwise couldn't be said before, or said only in an awkward, misleading fashion. In this respect, the introduction of this type of content is like the introduction of imaginary numbers to arithmetic, and the trick is to explore how such an introduction interacts with familiar, existing logical vocabulary, and how it might alter the familiar landscape of sentential, quantificational, and modal logics.

Fortunately, the introduction of such contents need not force any great revision to formal systems of proof. Here let's think of a familiar tree-system of logic, or "Tableau." In introductory texts, this method is often presented in sections devoted to semantics under the guise of "semantic" tableau (or the method of "truth trees"). As such, it is often presented in texts primarily as a device for quickly navigating truth tables in order to expose inconsistencies or to generate possible counterexamples, and so it is typically regarded as inherently tied to truth-functional semantics. Nevertheless, this very same method can also be given a ready interpretation in terms of the *pragmatic* attitudes of *commitment* and *preclusion* in an incompatibility framework.[7] The tree method works by developing the consequences of being *committed* to some assertible content, alongside the consequences of being *precluded* from certain commitments with the aim of exposing incompatibilities between patterns of commitments and preclusions. (In the more familiar "one-sided" trees, preclusion from a commitment is represented by commitment to that content's negation.) Intuitively, one can see why such a method would be commensurate with natural deduction systems deploying Gentzen- or Fitch-style elimination and introduction rules for the various sentential operators. The rules for developing commitments to certain contents obviously correspond to elimination rules, while the rules for developing precluded contents correspond to the introduction rules – except (as explicitly suggested by one-sided trees) that they work in the "opposite" direction. In essence, preclusion rules in Tableau, serve to "tollens" Gentzen-style introduction rules.[8]

With that in mind, let's spell out the rules for Tableau, now that we've introduced strikethrough contents. The rule for developing strikethrough contents on the left is straightforward: when a strikethrough is on the left, put both the formula stricken through and its negation on the right (see figure 1).[9] This captures the idea that the strikethrough of some content is so deeply incompatible with some content that it precludes both it and its negation.

Likewise, we need to introduce new rules for the preclusion of strikethrough contents. Basically, when a strikethrough is represented as precluded (or on the right-hand sight of a path), one develops all open paths on which it lies by branching that path and placing that which is struck through on the left of one branch and its negation on the left of the other (see the left-hand portion of figure 2).[10] We will also need to amend the negation rule on the right, so as to allow for

violations of the law of excluded middle. This is achieved in an entirely similar fashion. Negations on the right are developed by branching paths and placing that which is negated on the left of one branch, and its strikethrough on the left of the other.

Note that so conceived, these rules introduce an asymmetry between the pragmatic attitude of being committed to some content and being precluded from it. Specifically, the attitude of being committed to ~P is handled differently from the attitude of being precluded from P. For that reason, this system requires the deployment of "two-sided" trees rather than the "one-sided" variety one more often sees in introductory logic texts.[11]

Pleasantly, none of the other familiar Tableau rules need to be altered, which means that—except at the margins (literals) —this formal system (and the logic it supports) behaves classically.[12] It is also demonstrably sound and complete.[13] Of course, this completeness result does not mean that all classically derivable arguments will remain so, as attested by the invalidity of |= (P v ~P), and the corresponding failure of its being a theorem. Double Negation Elimination and the DeMorgan's equivalencies also fail to hold in this setting, specifically for literals. Indeed, one might well quip that this system is "strange" only at the atomic level. Since this system is evidently weaker than a purely classical system, it pleasantly follows that it is just as inferentially conservative as a purely classical logic. It won't introduce any run-about inference tickets.

So far, I've shown how one might begin to domesticate formally an attitude of rejection toward the direct application of a concept. To see how we might put strikethrough contents to good philosophical use, consider Dummett's well-worn example of a so-called rejected concept, that of a Boche. According to the received telling of the tale, this concept has introduction and elimination rules (or conditions and consequences of application) that effectively license inferences from someone's being of German descent to their being inhumane. The trouble with asserting that either somebody is a Boche or is not is that such an application effectively endorses the inference license embodied in the concept. To claim that I am Boche licenses one to conclude that I am inhumane, while to claim that I am not is to deny that I am of German descent. What are we to do?

The answer of course is to take Oscar Wilde's route and appeal to something like strikethrough contents. The correct attitude is to refuse to apply such suspect terminology in the current conversational context, in the precise sense of precluding the direct application of the rejected concept in claims that would be represented by stand-alone atomic formulas or their negations. Note that this is *not* to say that the rejected concept is altogether incoherent or nonsensical, or that we can't work with that concept in compound constructions. Indeed, it is the fact that we understand the concept all too well that prohibits our applying it directly to things in this world! But that doesn't mean that such a prohibition needs to carry over to other contexts accessed, for instance, through modal operations. While we

might refrain from applying a certain concept to things in this world (so to speak), that doesn't mean that we can't "think" in those terms, or that those terms can't be part of our language when we talk about various hypothetical or counterfactual contexts. One can quite well countenance scenarios in which a concept like Boche admits of direct application, just not "here."[14]

In a more tentative vein, let me suggest that the logical apparatus I've introduced here might be well-suited for supporting a novel response to the conceivability argument against materialism (aka the "zombie" argument). Just as the concept of Boche embodies the inhumanity of the Hun, it's not hard to see that the very concept of a philosophical zombie—of a minimal physical duplicate that lacks phenomenal consciousness—similarly incorporates an anti- or "super-" materialistic understanding of consciousness. So, it would seem that any materialist response ought to incorporate some rejection of the concept of a philosophical zombie.

Proponents of the conceivability argument (chiefly Chalmers) craftily demonstrate that the intimate connection we seem to bear to our own consciousness means that the bare conceivability of zombie-worlds dictates their metaphysical possibility, which in turn can seem to spell the end of the world for materialism. By way of reply, opponents have tried either to challenge the conceivability of zombies altogether or to block the inference from such conceivability to their possibility. Thus, they acknowledge that some sort of rejection of the zombie concept is in order. However, I would suggest that framing this rejection in terms of conceivability or possibility is not the best way to go. And indeed, these replies have met with limited success. The problem with the first strategy is the eminent conceivability of zombie worlds (distinct from our own); though perhaps ontologically repugnant, "super-"material worlds do seem at least epistemically possible. However, in order to take the second strategy by granting this weak sort of possibility while maintaining that they are not genuinely possible requires one to draw a satisfactorily sharp boundary between these two flavors of possibility. Usually, this difference is cashed out in a "two-dimensional" framework—that is, in terms of the difference between the "actual" denotation of some term and what that term "could" have denoted. Thus, the typical "type-B" rejoinder to the zombie argument is that it surreptitiously slides between what the concept of consciousness is actually applied to and what it could apply to in alternate realities. However, as Chalmers points out, the general problem with this sort of response is that the conception of consciousness animating the conceivability argument seems stubbornly univocal and rigidly attached to this world.

Still, the conceivability argument has an air of hocus pocus about it, and several recent commentators have suggested that its unsoundness is best exposed by considering matters, as it were, from the puzzlingly unstable perspective of a philosophical zombie.[15] For such a being would go through the same conceivability

exercises as we do and get things *wrong* (at least for it)! This strategy strikes me as much closer to the mark (and I've tried to pursue it elsewhere), but the trouble is that it considers matters from a stipulated hypothetical context. The question of materialism is really that of whether *we* turn out to be our own minimal physical duplicates. And if one has the concept of a philosophical zombie in their conceptual arsenal along with the license to apply it directly, then one can use that concept to argue that if materialism were true, then we would not be conscious, which of course is an absurdity.

The suggestion that I would tentatively float is that perhaps the best attitude to take here is to reject the super-materialist's own favored terms, specifically that of a zombie. Just as the proponent of the zombie argument wields the concept of a zombie to argue that none of us could be our own minimal physical duplicates, one might wield the concept of Boche (along with my evident humanity) to argue that I am not of German descent! By proscribing the zombie concept from direct application to us, we can perfectly well countenance a perspective from which zombies are both conceivable and possible, while at the same time holding that materialism is true and that we are our own minimal physical duplicates.

To be sure, it takes far more space than I have available here to flesh out this response.[16] Still, I hope to have rendered plausible the idea that the advent of strikethrough contents might provide us with some new ammunition in diagnosing the unsoundness of the conceivability argument. In this fashion, one can perhaps see that developing and elaborating a logic of rejection along the lines that I have started here, is a game well worth the philosophical candle.

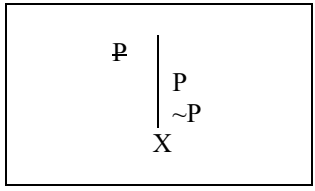
NOTES

1. My attitude towards truth (at least understood as a substantial property) might cheekily be summed up as follows: ~~Something is true~~. That is to say, truth is just not one of my go-to concepts.
2. And for what it's worth, I have tried to do a similar thing with a robustly modal conditional that behaves very much like the Lewis Hook, and from which a strong and weak modal operator can be defined.
3. For the uninitiated, one content incompatibility entails another just in case anything incompatible with the latter is also incompatible with the former. This notion of incompatibility entailment traces back to the stoic logician Diogenes.
4. In his Locke Lectures, Brandom (Published as *Between Saying and Doing*) seems to want to show that everything that can be done in a classical framework (including the modalities) can also be accommodated in an incompatibility setting. I am being a bit more ambitious here; I am introducing something in an incompatibility setting that cannot be accommodated in a classical one.
5. For this reason, we should not assimilate this approach to those, such as relevance logics, that introduce new truth values.
6. Brandom himself illustrates this idea through a discussion of Perry's "essential" first-person indexical.

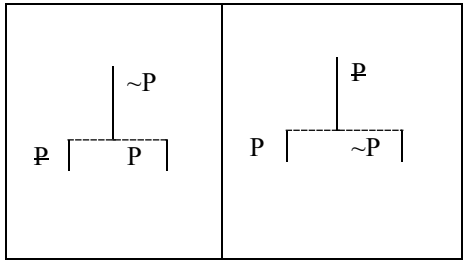
7. At the same time, the notion of logical validity can also be recast in terms of commitment and preclusion. A set of premises logically entails some conclusion just in case one cannot be coherently committed to all of the premises while also being precluded from the conclusion. Elsewhere (See my "Logic Beyond the Looking Glass"), I have shown how one might present formal logic as a theory of entailment without making any substantial appeals to the notion of truth.

8. With that in mind, one can readily discern what the Tableau rules for Belnap's infamous "Tonk" operator would be, even though it would be impossible to read such rules off of a truth table—namely the commitment rule of a conjunction, paired with the preclusion rule of a disjunction.

9. Figure 1: Tree development rule for strikethrough contents on the left.



10. Figure 2: Tree development rules for negation and strikethrough on the right.



11. Two-sided trees also have the advantage of being quite amenable to Rumfitt's "bilateral" semantics. Instead of the two sides representing the respective attitudes of commitment to or preclusion from various contents, they can just as easily correspond to Rumfitt's - attitudes of affirming a content (which he represents by the plus sign) and that of rejecting one (which he represents by the minus sign).

12. Although I shall not delve into the matter here, similar, "dual" rules can be introduced to capture paraconsistent logics. To do so, one would have to invent yet a different content, expressing the idea that one accepts *both* it and its negation. On the side of semantics, this would also require one to tweak one's understanding of the negation of some content, so that a content and its negation need not be understood as incompatible with one another as long as one also accepts the newly devised content expressing the attitude that both are acceptable. Developing such a semantics in an incompatibility framework would introduce an element of non-monotonicity, and would seem to require that one give up on the idea that incompatibility frameworks must obey a "persistence" condition. Once again, see Brandom (117-175).

13. On the soundness side, it is not hard to see that developing formulas according to these rules along a coherent path (one in which one may coherently be committed to all contents on the side corresponding to commitment while also being precluded from all the contents on the side corresponding to preclusion) is bound to leave at least one coherent (and open) path. On the completeness front, one can construct from a path that cannot be closed through suitably systematic development a fragment of an incompatibility frame according to which one may coherently be committed to all of the contents on the commitment side of that path while also being precluded from all the contents on the preclusion side of that path.

14. See Garson (2006) for a Tableau treatment of modal logic. This treatment also admits of an inferentialist-friendly interpretation in terms of commitment, preclusion, and incompatibility, though the details do get a little messy. One of the more philosophically intriguing things about strikethrough contents in a modal setting is that they might allow for the non-necessity of formulas such as a=a (in contexts where the term a is proscribed), while at the same time admitting the necessity of $(\forall x)x=x$.] In other words, they force us to examine more closely received Kripkean dogma about rigid designation and the necessity of identity claims.

15. I'm thinking here of recent papers by Balog, Stalnaker, Frankish, Yablo, Brown, and myself.

16. Specifically, I would suggest that when one lays out the argument in its canonical form (which makes no direct use of the concept of a zombie), the argument fails to distinguish the conceivability of a situation in which all the material facts are as they are and facts about consciousness (as the supermaterialist is apt to understand them) are *false* from the conceivability of a situation in which all the material facts are as they are and the qualitative facts expressed in the supermaterialist's preferred idiom are *rejected*. To be sure, I have neither world enough nor time to make that case stick. I only float this response as a suggestion.

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