EXISTENCE PROOFS AND THE ONTOLOGICAL ARGUMENT

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What might be called the generic criticism of the ontological argument—that existence cannot be formally demonstrated—often occasions a rejoinder which is seemingly effective. This rejoinder consists in pointing out that both logic and mathematics make use of what are called 'existence proofs.' It is claimed, for example, that we can prove that the round square does not exist. The argument seems to suggest that if logicians and mathematicians may prove and disprove existence with impunity, then it is unfair to bar theists from doing likewise. Despite this, I want to argue that the advocates of the ontological argument can draw no legitimate support from the consideration of existence proofs in logic and mathematics. Further, I want to suggest that insofar as the idea of necessary existence is drawn from logical notions, it rests on a confusion. I appreciate the veneration of logic on the part of theists which this attempt at support implies, but I still insist that logicians be wary of any co-optation of their devices for the purpose of propping up superstition.

Lest it be suspicioned that I am taking aim at a straw man, consider the words of G. E. Hughes "the proposition 'there is a round square in the next room' could be refuted by an ontological disproof."[1] What Hughes no doubt has in mind is an argument which claims that since the concept of a round square is self-contradictory, there could not possibly be anything in the next room which would count as a round square.

Such reasoning, though common, is misleading in the following way. The person who puts forward an ontological disproof of the sort in question draws an implicit comparison between round squares and the normal four-sided variety. He then infers that since the concept of a round square is self-contradictory while the concept of a normal square is not, there must be a corresponding difference in the ontological status of round squares and normal squares. But what could this difference be? It cannot be that one is found in rooms and the other is not, for squares (using the term in its technical or geometric sense) are not spatial objects, and are therefore no more likely to turn up in rooms than are unicorns. In fact, once we realize that no square of any sort can be found in a room, it follows trivially that no round ones can be found.

The proponent of ontological disproofs might attempt to evade the impact of the point made above in two ways. The first strategy he might employ would involve the following shifting of his ground. It is true, he might admit, that in the technical sense, neither round squares nor regular ones are found in rooms. However, the distinction between the two can be saved by considering their non-technical meanings. We can find squares (in the everyday sense of 'squares') drawn on blackboards, but not so in the case of round squares. Unfortunately, this strategy fails by giving up too much. Ordinary meanings cannot support the concept of logical self-contradiction needed to make an ontological disproof work. It is nonsensical to say that anything which can be drawn on a blackboard is logically consistent. So we are still lacking an ontological difference between round squares and regular ones.

The second strategy would go somewhat as follows: It is true that neither round squares nor regular squares are spatial objects, but this is beside the point since it is not spatial existence which is at issue. Regular squares exist in some (non-spatial) realm not occupied by round squares. Two responses may be made to someone who takes this tack. First, if it is not spatial existence which is at issue, then the example is misleading. In our unreflective thinking we tend to picture squares as things which can be drawn on blackboards, paper, and so on. It is only because we do this that the difference between round squares and regular squares strikes us as having ontological significance. Of course, this remark would not apply to different examples, but the example of the round square is so common as to be almost standard.

Second, it does not follow that because we are not discussing spatial existence, we must, therefore, be discussing existence of another sort. The disprover of round squares would have us believe that his purported proof tells us something about ontology, but he fails to show what this might be. He may believe that round squares belong in an ontological never-never land, but he has not demonstrated that they differ in any ontologically significant way from ordinary squares.

If the fact that the concept of a round square is self-contradictory while the concept of a square is not does not reflect an ontological difference, what sort of difference does it reflect? Well, there is a difference to be sure, but I am afraid it is not nearly so dramatic as the proponents of ontological proofs and disproofs would have us believe. To say that the concept of a round square is self-contradictory is merely to say that within the structure of a certain formal system (i.e., geometry) the terms 'round' and 'square' cannot be concatenated. And in case someone is tempted to respond by saying that the concept of a round square is not only disallowed in geometry but is *really* self-contradictory, I would remind him that the idea that geometry describes real spatial objects was abandoned some time ago. Furthermore, it will not do to disclaim the formal system since, as was pointed out above, only a formal system can supply the concept of self-contradiction needed to support an ontological disproof.

Perhaps the point I am trying to establish will be clearer if we consider an example from a different area of mathematics. Instead of talking about round and regular squares, let us consider a "square" of a different sort—the square root of two. This example is not subject to the confusion which can arise as a result of spatial interpretation. Does the square root of two exist? As in the earlier example, there appears to be a question of ontology involved, but this appearance is again misleading. Consider the following propositions:

Q: $\sim$ E x $(x \cdot X = 2)$.
R: E x $(x \cdot X = 2)$.

If we take Q and R to be expressing metaphysical truths, then one of them must be false since they are contradictories.

However, it is doubtful that any mathematician would see them as contradictories. Instead, he would see them as theorems of two different systems. Q is a theorem of the rational number system; whereas, R is a theorem of the real number system. But what do we say to someone who insists on a yes or no answer to the question of whether the square root of two exists? We would have to explain that no such answer can be given. If we choose one set of axioms (or assumptions) we can show that the square root of two exists; but if we choose a different set of axioms, we can show that it does not exist.

Although it is standard practice to read "E x" as "there exists an x . . . ," no ontological commitment is implied. If we want to be completely precise in our reading of R, we should not say that something exists but that, given the axioms of the real number system, the proposition R logically follows. Since there is no reason to believe that the real number system is somehow "truer of reality" than the rational number system, we should stop short of claiming ontological significance for R.

Further evidence that the backwards E does not assert existence in any ontologically significant way is provided if we accept the substitution interpretation of quantifiers. According to this interpretation, E x(Fx) is true iff there is a term Fa such that Fa is true. Now consider E x(Ux) where U stands for the property of being a unicorn. The formula in question does have one true substitution instance, namely, "unicorn = unicorn." Hence, no ontological significance can be given to the existential quantifier on pain of granting existence to everything. Of course, the proponents of ontological proofs need not accept this interpretation, and to argue its merits

is beyond the scope of this paper. Suffice it to say that the substitution interpretation has a lot to recommend it.

If what I have just argued is correct, then the most that the proponents of the ontological argument could claim is that the expression, E x(Px), where P stands for the property of being perfect, follows from certain assumptions, viz., the premisses of the argument and the logical truths of the system of logic employed (usually Lewis's S5). Now, it is easy to find or construct a system of logic in which the proof in question does not hold, so that the expression E x(Px) is similar in this respect to the expression which states that a square root of two exists. Either of these expressions follow from some sets of assumptions and not necessarily from other sets. From this consideration we must conclude that no ontological truth can be inferred from either.

It seems to me that there is a growing recognition on the part of logicians and mathematicians that their fields are paradigmatically studies of the relations between propositions. The mathematicians of today are less likely to view their work as the elucidation of a particular aspect of reality (e.g., numbers) than were their predecessors. In logic, the advent of "free logic" and similar systems demonstrates a desire to escape the bramble of ontological commitment. But despite the fact that the points I have made thus far are sometimes implicit in the attitudes of working logicians, they deserve explicit formulation so long as there are those who insist that logic can provide us with metaphysical truth.

Let me now turn to the notion of necessary existence. As before, I shall start with an example from mathematics before turning to the ontological argument. We have already noted that the existence of the square root of two can be proved in the real number system (I have no aversion to the use of the word 'existence' in this context so long as we do not infer ontological commitment). Does the square root of two necessarily exist? The answer must be a qualified yes. If we mean no more than that the theorem R follows necessarily (logically) from the axioms of the real number system, then, yes, the square root of two necessarily exists.

But the advocates of the ontological argument want to claim more than this about the conclusion of their argument. It is not the proposition which they want to be necessary, nor its entailment by the premisses. Instead, it is the being itself and/or its existence which is declared to be necessary. But there is nothing in formal logic which corresponds to necessity in this sense, and the necessity operator is no exception. In the expression, $\Box$ E x(Px), the square modifies the proposition E x(Px), not the variable x or the existence of x. It is, therefore, a mistake to think that the ontologists' concept of necessary existence is drawn from logic.

The concepts of existence and necessity are certainly not confined to logic, but their use in logic is ontologically neutral.

NOTE

1. "Can God's Existence be Disproved?" in *New Essays in Philosophical Theology* (New York: The Macmillan Company, 1955).