Ballots in the Belfry: Lewis Carroll and Voting Fairness

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The following paper has been written and printed in great haste, as it was only on the night of Friday the 12th that it occurred to me to investigate the subject, which proved to be much more complicated than I had expected. Still I hope that I have given sufficient thought to it to escape the commission of any serious mistake.

—Lewis Carroll, "A Discussion of the Various Methods of Procedure in Conducting Elections"[1]

The following paper has been written in great haste, as it occurred to me shortly before the deadline for submission that this might be the most timely year to investigate Lewis Carroll's concern for voting fairness.

The concern over the fairness of the 2000 presidential election has focused on the construction of ballots, voting machines, the little chad (a small country in the Florida portion of Africa, I believe), voting recounts, and the Electoral College (a small unaccredited college back east somewhere, I think). What hasn't received attention is the fairness of various systems of voting. What hasn't received attention is the fact that there even are different basic systems of voting.

Voting fairness was a topic with which Lewis Carroll was concerned and on which he produced some pamphlets. It is a major topic in the field of study known as voting theory.

Voting theory is the branch of mathematics that deals with the process by which democratic groups resolve the differing opinions of the members of the group into a single choice of that group. This field is a relatively new field and an examination of any historical account of voting theory shows the earliest contributions to be from Jean-Charles de Borda (1733-1799) and the Marquis de Condercet (1743-94). But the next person mentioned is likely to be C. L. Dodgson (Lewis Carroll., 1832-1898). The flowering of the field did not come until the 1950s with the work of Duncan Black, Kenneth Arrow, Lloyd Shapley, Martin Shubik, and Robin Farquharson. (Black, writing about his early work in the field, says, "Dr Newing suggested to me the use of a matrix notation which, some years afterwards, I discovered had already been employed by Rev. C. L. Dodgson . . . ."[2])

As philosophers, the first question that pops into our minds might be the question of what we mean by "fairness." Voting theorists have considered a

number of elements that fairness might involve. Two that are relevant in most cases we encounter are the majority criterion and the Condorcet criterion. The majority criterion holds that any candidate receiving a majority of first place votes should be the winner. The Condorcet criterion holds that a candidate who wins head-to-head match-ups with all other candidates should be the winner.

Fairness of the voting procedure is not a problem if there is only one candidate; however, some might object to a dictatorship on other grounds. Fairness of the voting procedure is not a problem if there are only two candidates; however, some might object to the race involving two candidates, both of whom they loath. If "neither of the above" becomes a third option or if a third candidate enters the race, the issue of fair voting procedures rears its important head.

Suppose that in an election between Bill and Bob, Bob would win by a small margin. But if Ross enters the race, he would attract very few voters but just enough of the voters who previously preferred Bob to allow Bill a close win. How can this type of election be fair if the outcome (between Bill Clinton or Bob Dole) is determined by a third candidate that has no realistic chance of winning? And if nothing else it leads to "faithless" voters voting for the candidate Bob that they don't prefer (but is their second choice) and who end up feeling untrue to themselves as well as having their least-favorite candidate winning. If Bob wins with just a plurality of the popular votes this certainly violates the majority criterion. Since voters don't fill out ballots where they rank all candidates against each other, we cannot know if the Condorcet criterion was met, but it probably was given what we know from surveys.

Suppose that in an election Norm and Hubert are defeated by Jesse. Jesse won because he was a refreshing outsider who attracted many voters who would not have otherwise have bothered to vote for anyone. But if Jesse is a political neophyte with little education and little knowledge of important issues, and if the vast majority of the voters desperately wanted an experienced leader and would have ranked Jesse third, the result might seem even to an outsider to be unfair. But Jesse Ventura served as Governor of Minnesota and Spokesperson for Unusual Forms of Entertainment.

Even if in the current election George had garnered a plurality of the popular votes, he would still have been the last choice (out of three candidates, ignoring other parties) of the majority of voters. As Michael Kinsley says, "so-called 'majority rule' rarely gives a majority of voters their first choices. What we really have is 'plurality rule.' In the past three presidential elections, no candidate got a majority of the vote. Most voters' preferences were frustrated."[3] Of course it turned out to be even worse since President George W. Bush did not even gain a plurality of the popular votes.

presidential system and most public voting systems, and can't indicate a ranking of other preference, a lot of important information is lost. So, not only have we as academic philosophers been thinking about how our state and national elections should be handled, we have probably harbored worries about how we have conducted voting for new faculty hiring, tenure and promotion, deanships, and so forth.

Lewis Carroll's interest in voting was certainly primarily because of his participation and interest in these kinds of college issues. He was an inveterate pamphleteer and three of these are about voting.[4] The first, "A Discussion of the Various Methods of Procedure in Conducting Elections," was written in 1873 just in time for an election (for the Lee's Readership in Physics) in his college, Christ Church, Oxford, at which he says, "we partly used my method."[5] In this work he tries first to show that a number of systems of election are flawed.

Here, Carroll's first case and first method ("simple majority" by which he obviously means the method of plurality) is illustrated by eleven voters voting for *a*, *b*, *c*, or *d*:

Chart I

a a a b b b b c c c d

Carroll says *b* wins by this method. Carroll's table, however, shows how each voter would rank each candidate against the others (This today is called a preference ballot.):

Chart I

a a a b b b b c c c d
c c c a a a a a a a a
d d d c c c c d d d c
b b b d d d d b b b b

The additional information is startling. Carroll asserts that *a* should be elected since *a* ranked first by three of the electors and second by all the rest, while *b* only got one more first-place ranking and was rated worst by seven of the voters.

The second method is the method of majority vote which he calls "absolute majority."

Chart II

b b b b b b a a a a a
a a a a a a c c c d d
c c c d d d d d d c c
d d d c c c b b b b b[6]

Carroll points out that *b* wins by this method, but there is "no doubt that *a* ought to be elected"[7] because *a* is one vote short of a majority and considered to be second by the rest of the electors, while *b* is considered to be the worst by five

electors.

The third method Carroll considers and rejects is "The Method of Elimination, where the names are voted on by two at a time." Here two candidates (or issues) are voted on and the winner pitted against another of the candidates until every candidate has been subject to vote. This method has the "preposterous" result of "making the Election turn on the mere accident of which couple is put up first...."[8]

Considered next is "The Method of Elimination, where the names are voted on all at once." The person with the fewest votes is eliminated and everyone votes again, and this is repeated until the winner is determined. But Carroll shows that a candidate could win by this method even though being the last or next to last choice of the majority of the voters while a losing candidate might be the first or second choice of all the voters. Carroll thinks this result is unfair.

After examining these four systems, Carroll considers the "method of marks," a method in which a number of votes are assigned each elector and these may be assigned in any manner whatsoever. Ideally this allows each elector to record exactly his or her assessment of each candidate or issue to be voted upon. But as Carroll notes, "we are not sufficiently unselfish and public-spirited to give any hope of this result being attained."[9] Carroll knows that human nature is such that voters will often give all their marks to their favorite to lessen the chance that others may win even if some other candidate is thought to be almost as good. The result, he says "is therefore liable, in practice, to coincide with 'the Method of a Simple Majority [plurality]', which has been already discussed, and, as I think, proved to be unsound."[10] So, his "proposed Method of Procedure"[11] is, he says, a modification of the method of marks. Each voter ranks each candidate in order of preference, the least favorite receiving no "marks" or points, the next least favorite receiving one vote, and so on up to the favorite who will receive points equal to one less than the total number of candidates. To prevent essentially the same problem as just discussed, if the elector wants to rank all but the favorite together ("in order to reduce their chances as much as possible"), they must all be given "the same mark that the highest would have if the bracket were removed."[12] Carroll then shows by adding up the marks that this method gives what he considers the fair result in all the cases he has previously considered. This is essentially what is currently called the Borda Count Method. (In this work he also treats "no election" as if it were one of the candidates.)

After the first work ("A Discussion of the Various Methods....") had been published, he decides that an even better method involving head-to-head comparisons of pairs of candidates, issues, or proposals should be used, if possible. A new belfry was needed and there was wide difference of opinion about what it should look like. Carroll was interested in the belfry design and of

course was thinking about fair elections, but his election ideas were not fully worked out. But in anticipation of an upcoming meeting about the belfry he published the very short (about 700 words) "Suggestions as to the Best Method of Taking Votes, Where More Than two Issues Are to Be Voted On." (His suggestions are essentially what is now known as the Condorcet criterion or the method of pairwise comparison.) Carroll just states the procedure he thinks should be used and provides no explanation or justification for his system. And in reference to his earlier pamphlet he only says he has "since seen reason to modify some of the views therein expressed."[13]

But it turned out that this was just an emergency loan against future argument and a couple of years later he published the third pamphlet, "A Method of taking Votes on More Than two Issues."[14] This pamphlet is by all accounts quite remarkable. It has received praise from mathematicians and voting theorists alike.[15] It is in this work that Carroll develops the matrix notation that was mentioned earlier in this paper. Let's take the following example from Carroll of fifteen voters' rankings of four candidates:

Chart III

a	a	a	a	b	b	b	b	c	c	c	c	d	d	d
d	d	d	d	c	c	c	c*	d	d	d	d	a	a	b
b	b	b	b	d	d	d	d*	a	a	b	b	c	c	c
c	c	c	c	a	a	a	a	b	b	a	a	b	b	a

Carroll examines the result using the rule "That all candidates should be voted on at once, and the one who has the smallest number of votes should be struck out, and the process repeated till only two are left."[16] We can do this by examining the table above if we assume no one's preferences would change in subsequent votes. *d* gets the fewest votes initially and would be eliminated. Imagine all other candidates moving up if an elector had initially preferred *d* over that candidate. In the second round *a* picks up two votes and *b* picks up one, and *c* would drop out. In the third round *a* and *b* each pick up two votes giving the win to *a* with eight votes to just seven for *b*.

However, it is here that Carroll begins to get to the important part of his paper. He objects to this result. He points out that in looking at how each possible pair of candidates fare against each other we discover what are called cyclical majorities. This can be worked out on a table to show who wins each head-to-head match-up:

Chart IV

	a	b	c	d
a	-	a	c	d
b	-	-	b	d
c	-	-	-	c
d	-	-	-	-

Candidate *a* is preferred to *b*, but *b* is preferred to *c*, *c* is preferred to *d*, but *d* is in turn preferred to *a*! Carroll introduces his matrix to show the results of the head-to-head match-ups including the number of votes each candidate gets. To read the matrix below you must note that you read down the columns. For example, the item I've marked with an asterisk shows that in considering the match-up between *d* and *c*, *d* was preferred to *c* by seven voters while *c* was preferred to *d* by eight of the voters. You will find the fraction inverted if you look down the *c* column to the item with the pound symbol.

Chart V

	a	b	c	d
a	-	7/8	9/6	11/4
b	8/7	-	6/9	11/4
c	6/9	9/6	-	7/8*
d	4/11	4/11	8/7#	-

But Carroll discusses how many changed votes it would take for a candidate to win by what is now called the Condorcet criterion, that is to win every head-to-head match-up. If we look in column *a*, we see six changes would be needed. *a* would have to get two changes from those preferring *c* and four changes from those preferring *d*. For *b* to win five changes would be needed, one from the *a* voters, and four from the *d* voters. For *c* to win two changes would be needed, but for *d* to win only one change would be needed. Looking back at the two items marked with an asterisk in Chart III one can see if the eighth voter had simply preferred *d* to *c* rather than the other way around, *d* would have been the winner by the Condorcet criterion. Carroll believes that *d* "ought to" win.[18] However, he does not say we should declare *d* to be the winner. In light of these results, Carroll thinks voters ought to be allowed to consider changing their votes. It seems obvious that Carroll thinks that if any one is to win at this point it is most likely to be *d*. But, he concludes, "... in the case of persistent cyclical majorities, there ought to be 'no Election.'"[19]

So, in these three small works we see Carroll's obvious concern with fair voting supplemented by the logical-mathematical bent of his mind at work on both real and imagined examples of voting results. We see his ideas being implemented in actual college elections and his conclusions changing slightly over a period of a few years. A conclusion he never strayed from was the unfairness

of declaring a winner by a mere plurality. I think he is obviously correct and this is shown not only by his examples and the Jesse Ventura example, but just recently by the John McCain candidacy for the office of President. McCain's favorable rating was higher than any other candidate for president in early February 2000 and widened during the month until on February 28th he had an approval rating of 66 percent to ratings of 59, 57, and 54 percent for Al Gore, George W. Bush, and Bill Bradley, respectively. Yet, because our system of primaries is based on plurality wins, McCain was out of the running just seven days later! Unfortunately the intrinsic problem with plurality systems is not even solved by a run-off when there are large numbers of candidates because the candidates that make it into the run-off can do so with very little overall support while candidates who would be acceptable to wide numbers of voters can be left off the run-off ballot.

Carroll's objection, in his first pamphlet, to the majority candidate winning the election (See Chart II, above) is much more controversial, and Carroll seems to have abandoned his claim when he says, "If the Chairman find any issue having an absolute majority of votes ... the Chairman shall declare it carried."[20]

As we saw, Carroll in his first paper advocated his modified method of marks, which is essentially the Borda system. This system has some strong advocates today, and the best known may be Donald Saari. The advantage of this system is that allows the use of much more information about voter attitudes and it prevents the kind of result that Saari describes by saying, "The plurality vote is the only procedure that will elect someone who's despised by almost two thirds of the voters."[21]

Carroll abandons his method of marks in the last two pamphlets because, I think, a winner is not approved by a majority if considered in head-to-head competitions with each other candidate.[22] His concern about cyclical majorities and what we would now call the Condorcet criterion leads him to want to require that for a winner to be declared the candidate must win all pair-wise match-ups. It is to this end that he wants to announce to the electorate the number of vote changes it would take to produce a winner in the hopes that a re-vote will produce exactly that. Failing this, there is no election.

Clearly "no election" is often not an option in elections today. So, I will close will a consideration of practicality. A weakened Condorcet system would give the election to the candidate who wins the most head-to-head battles. And almost always the winner by Borda and Modified Condorcet will the same. There is then little reason to prefer one of these over the other. We might choose either in our election at home and at work. However, they both fail a test of practicality in cases like our national elections. We have no provision for a voter to rank all the candidates against each other and it would be extremely difficult to implement such a system. Up to [illegible] fail

record voters first-place choices.

I will, therefore, briefly introduce and recommend for your consideration a system that is usually called Approval Voting. Possibly its strongest advocate is Steven J. Brams.[23] Stated simply, it allows a voter to cast one vote for any and every candidate deemed satisfactory. One could vote for just one candidate or for more than one. The winner is the candidate with the most approval votes. Notice that this is easy to implement in any of the current voting mechanisms, punch card, machine-readable pencil, or hand-counted ballots. *If* a voter actually approved of Buchanan and Gore, the ballot would look like one of those infamous Palm Beach County, Florida, ballots in 2000. But, more likely, there would have been many people who would have approved of both Gore and Nader. This would have resulted in a win for Gore who was, of course, the voters' favorite. Or, in 1992, voters could have approved of both Perot and Bush, giving the win to George Bush, the elder. (See, there is something here for everyone.) Here is a summary of the merits of approval voting[24]

1. It gives voters more flexible options. A voter can vote for one or more than one candidate.

2. It is relatively insensitive to the number of candidates running. Additional candidates don't have to draw votes from other candidates, but can be additionally approved by the voter.

3. It helps elect the strongest candidate. The most strongly approved candidate would win instead of the candidate with the largest minority.

4. It increases voter turnout. Persons will go to the polls in greater numbers because they are better able to express their judgments.

5. It gives minority candidates their proper due. Voters with small followings will be able to receive votes from those who approve of them but who otherwise would vote for a candidate thought to have a better change of winning.

6. It will add legitimacy to the outcome. Candidates who under the present system might win without a majority of votes, could often receive a majority under approval voting.

7. It is very practical. It is easily implemented. It is more "strategy-proof" than other systems. It, in most cases, would only require statutory, not a constitutional, change.

Plus, it reduces negative campaigning. Candidates will have a reason to attempt to broaden their appeals by reaching out to voters who might have a different first choice—without alienating them. Unfortunately a detailed defense of these claims is not possible here, but I hope we will think about alternative voting systems with an eye to fairer elections.

Finally, before we leave here today to ensure that our time here has been well spent, I hope you and I can make a definitive judgment about which of these many systems is the fairest. Let's vote on it.

NOTES

1. In fact, the preface is dated Dec. 18, 1873, just 6 days after his diary tells us he began working on the pamphlet.
2. Duncan Black, *The Theory of Committees and Elections* (Cambridge: Cambridge University Press, 1958), p. xi.
3. Michael Kinsley, "Democracy is Approximate," *Washington Post*, Nov. 16, 2000, p. A43.
4. All three of them are printed in an appendix to Black's book, pp. 214-234.
5. *The Diaries of Lewis Carroll*, edited and supplemented by roger Lancelyn Green, in two volumes (London: Cassell & Co., Ltd., 1953), diary entry Dec. 18, 1873.
6. "A Discussion of the Various Methods of Procedure in Conducting Elections," in Black, p. 216.
7. Black, p. 217.
8. Black, p. 217.
9. Black, p. 218.
10. Black, p. 218.
11. Black, p. 221.
12. Black, p. 221.
13. Black, p. 222.
14. It contains an explicit promissary note that we could say was partially paid off by his *The Principles of Parliamentary Representation*. "... I hope to investigate this subject further, and to publish a more complete pamphlet on the subject ...." Black, 224. C. L. Dodgson, *The Principles of Parliamentary Representation* (London: Harrison and Sons, 1884). Dodgson (Carroll) also published *The Principles of Parliamentary Representation: Supplement* (Oxford:, 1885)
15. Francine Abeles in her article "The Mathematical-Political papers of C. L. Dodgson," in Edward Guiliano, ed., *Lewis Carroll: A Celebration* (New York: Clarkson N. Potter), p. 201, says that in these pamphlets "Dodgson showed a grasp on the intuitive level, of ideas that did not begin to be formalized until the 1920s." She also says (p. 190) that in the part of the third pamphlet involving the matrix Dodgson "was really describing a ... method ... called ranking by inversion, and the model that it anticipated, a maximum-likelihood weak stochastic rank order ..., first described by Walter Thompson and Russell Remage, Jr., in 1964. Also see Duncan Black's comment in Black, p. 189, where he mentions "the importance of Dodgson's work."
16. Black, p. 228. Italics removed.
17. Black, p. 229. Italics mine.
18. Black, p. 224.
19. Black, p. 230
20. Black, p. 224
21. Quoted in Dana Mackenzie, "May the Best Man Lose," *Discover Magazine*, Vol 21, No. 11, p. 84. Mackenzie also notes another example of the perils of plurality in the 1991 Louisiana governor's race in which the incumbent governor Buddy Roemer does not make it to the run-off. The run-off pitted former grand wizard of the KKK David Duke (32 percent of the primary vote) against the often-indicted Edwin Edwards (34 percent) because the incumbent Buddy Roemer only garnered 27 percent of the vote. Edwards

wins (And Roemer supporters had to "vote for the crook.") although it unlikely he would have won in a Borda election.

[22.] Another objection to the Borda Count system is that it is prone to attempts to beat the system by not ranking one's choices in the order that they are actually perceived by the voter. Even Borda himself was aware of the problem, saying, "My system is only for honest me." Quoted by Mackenzie.

[23.] See Steven J. Brams and Peter C. Fishburn, *Approval Voting* (Boston; Basal: Stuttgart: Birkhäuser, 1982).

[24.] From Brams and Fishburn, pp. 3-10.